An AP scheme for the Fokker-Planck-Landau equation based on penalization

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Abstract

We present an AP scheme for the (rescaled) nonhomogeneous Fokker-Planck-Landau (nFPL) equation,

$$\partial_t f + v \cdot \nabla_x f = \frac{1}{\varepsilon} Q(f), \qquad x \in \mathbb{R}^{N_x}, v \in \mathbb{R}^{N_v}$$

with

$$Q(f) = \nabla_v \cdot \int_{\mathbb{R}^{N_v}} A(v - v_*)(f(v_*)\nabla_v f(v) - f(v)\nabla_{v_*} f(v_*))dv_*,$$

where $A(z)$ is a semi-positive definite matrix and ε is the Knudsen number.

An explicit scheme for this equation requires $\Delta t = O(\varepsilon \Delta v^2)$ due to the diffusive nature of $Q(f)$. An implicit scheme has no such restriction on the time step. But implicit schemes involve inverting an operator containing $Q(f)$, which cost a lot if one uses Newton's solver.

In this paper we design an implicit-explicit type scheme based on the penalization by another operator, the linear Fokker-Planck (FP) operator. This is inspired by the recent work of Filbet and Jin, on the numerical scheme for the classical Boltzmann equation. In this new scheme, Δt is only constrained to the CFL condition from the transport part. And we only need to invert a linear system involving the FP operator, instead of the nonlinear one, which involves the nFPL operator. We also design a new discretization of FP operator so that one obtains a symmetric matrix, which can be solved efficiently. Besides the scheme is able to capture the macroscopic system when $\varepsilon \to 0$. Therefore it is Asymptotic-Preserving (AP). Numerical experiments are also carried out to verify the performance.