DISPERSIVE ESTIMATES FOR MATRIX SCHRÖDINGER OPERATORS IN DIMENSION FIVE

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The non-selfadjoint matrix Schrödinger operator,

$$\mathcal{H} = \begin{bmatrix} -\Delta + \mu - V_1 & -V_2 \\ V_2 & \Delta - \mu + V_1 \end{bmatrix}$$

arises when linearizing about a standing wave solution in certain non-linear Schrödinger equations. We investigate the boundedness of the evolution operator $e^{it\mathcal{H}}$ in the sense of $L^1 \to L^\infty$ in dimension five where $\mu > 0$ and V_1, V_2 are real-valued decaying potentials. In particular, we show that under standard spectral assumptions on $\mathcal{H}$, if $V_i \in C^1(\mathbb{R}^5)$ with $|V_i(x)| \lesssim \langle x\rangle^{-4-}$ and $|\nabla V_i(x)| \lesssim \langle x\rangle^{-3-}$, then

$$\|e^{it\mathcal{H}} P_c\|_{L^1 \to L^\infty} \lesssim |t|^{-\frac{5}{2}},$$

where P_c is projection away from the eigenvalues of $\mathcal{H}$.

This work builds on the five (and seven) dimensional result of Erdoğan and Green for the scalar case, which proves a similar theorem for the scalar Hamiltonian $H = -\Delta + V$. This new result improves the scalar result by reducing the decay requirement on the potential from $|V(x)| \lesssim \langle x\rangle^{-10-}$ to $|V(x)| \lesssim \langle x\rangle^{-4-}$.