Sample Final 2008

1. Suppose $z = f(x, y)$, $f_x(a, b) = 0$, $f_y(a, b) = 0$, $f_{xx}(a, b) = 16$, $f_{yy}(a, b) = 16$, and $f_{xy}(a, b) = 16$. Then (a, b) is

A) the test is inconclusive
B) a saddle point
C) a relative minimum
D) a relative maximum

2. The only grocery store in a small rural community carries two brands of frozen apple juice, a local brand that it obtains at the cost of 18 cents per can and a well-known national brand that it obtains at the cost of 60 cents per can. The grocer estimates that if the local brand is sold for *x* cents per can and the national brand for *y* cents per can, approximately 70 – 5x + 4y cans of the local brand and 80 + 6*x*-7*y* cans of the national brand will be sold each day. How should the grocer price each brand to maximize the profit from the sale of the juice?
Rather Long (JL). Construct a Total Revenue and find its first and second partial derivatives.

A) local brand (*x*) at 74 cents, national brand (*y*) at 55 cents
B) local brand (*x*) at 32 cents, national brand (*y*) at 114 cents
C) local brand (*x*) at 39 cents, national brand (*y*) at 57 cents
D) local brand (*x*) at 37 cents, national brand (*y*) at 57 cents

3. Compute f_{yy} for $f(x, y) = e^{4xy}$.

A) $16x^2e^{4xy}$
B) $16y^2e^{4xy}$
C) $16e^{4xy}$
D) $16x^2y^2e^{4xy}$

4. Find the second partial f_{xy} given $f(x, y) = 5xe^{8xy} + y\ln 2x + 9y$

A) $18(1+x)e^{8xy} + \dfrac{-18xy}{(2x+9y)^2}$
B) $40x(2+8xy)e^{8xy} + \dfrac{4x}{(2x+9y)^2}$
C) $18(1+x)e^{8xy} + \dfrac{-18x}{(2x+9y)^2}$
D) $40x(2+8xy)e^{8xy} - \dfrac{4x}{(2x+9y)^2}$

5. Find the second partials (including the mixed partials) of $f(x, y) = 7x^5y^6 + 5xy$

A) $f_{xx} = 140x^3y^6$, $f_{xy} = 210x^4y^5 + 5$, $f_{yx} = 210x^4y^5 + 5$, $f_{yy} = 210x^5y^4$
B) $f_{xx} = 140x^4y^6$, $f_{xy} = 210x^5y^5 + 5$, $f_{yx} = 210x^5y^5 + 5$, $f_{yy} = 210x^5y^5$
C) $f_{xx} = 140x^3y^6 + 2y$, $f_{xy} = 210x^5y^5 + 5$, $f_{yx} = 210x^4y^5 + 5$, $f_{yy} = 210x^5y^4 + 5x$
D) $f_{xx} = 140x^3y^6$, $f_{yy} = 210x^5y^4$

6. Compute f(ln2, ln7) if $f(x, y) = e^{2x+y}$.

A) 28
B) 14
C) 11
D) none of the above

7. Let $f(x) = 10x^9 - 180\ln x$, for $x > 0$. Find the minimum value of *f* for $x > 0$.

A) $5(2^9 - 18\ln(2))$
B) $10(2^9 - 18\ln(2))$
C) 0
D) 20(1 – ln(2))

8. Suppose your family owns a rare book whose value *t* years from now will be $V(t) = 4e^{\sqrt{5t}}$ dollars. If the prevailing interest rate remains constant at 6% per year compounded continuously, when will it be most advantageous for your family to sell the book and invest the proceeds?

A) 277.78 years
B) 347.22 years
C) 416.67 years
D) 486.11 years

9. How quickly will money triple if it is invested at 7% interest compounded continuously?
A) 15.69 years
B) 15.71 years
C) 15.73 years
D) 15.75 years

10. Find $\frac{df(x)}{dx}$, where $f(x)=\ln x^6$.
A) $\frac{6}{x}$
B) 6*x*
C) $\frac{x}{6}$
D) $\frac{1}{6x}$

11. The equation of the tangent line to $f(x)=8\ln x^3$ at *x* = *e* is
A) $y=\frac{24}{e}x$
B) $y=\frac{24}{e}x+48$
C) $y=\frac{24}{e}x-48$
D) *y* = 24*x*

12. Find the derivative of $\ln\left(\left(\ln x^2\right)^3\right)$
A) $\dfrac{3}{x\ln x}$
B) $\dfrac{6}{x\ln x}$
C) $\dfrac{6}{x+\ln x}$
D) $\dfrac{6}{\ln(\ln x)}$

13. The consumer demand for a certain commodity is $D(p)=7000.00e^{-0.52p}$ units per month when the market price is *p* dollars per unit. Express consumers'total monthly expenditure for the commodity as a function of *p* and determine the market price that will result in the greatest consumer expenditure.
A) \$28.85
B) \$7000.00
C) \$1.92
D) \$15.38

14. Use logarithmic differentiation to find $f'(x)$. $f(x)=\sqrt[4]{\dfrac{6x+9}{4+4x}}$
A) $f'(x)=\left(\dfrac{6}{6x+9}-\dfrac{4}{4+4x}\right)f(x)$
B) $f'(x)=\dfrac{1}{4}\left(\dfrac{6}{6x+9}-\dfrac{4}{4+4x}\right)f(x)$
C) $f'(x)=\left(\dfrac{6x+9}{4+4x}\right)^{-3/4}$
D) $f'(x)=f(x)\left(\dfrac{6x+9}{4+4x}\right)^{-3/4}$

15. Solve for *x*: $5\ln x-\frac{1}{7}\ln x^5=30$.
A) $x=e^7$
B) *x* = *e*
C) $x=e^{30}$
D) $x=e^5$

16. A radioactive substance decays exponentially. If 600 grams were present initially and 200 grams are present 100 years later, how many grams will be present after 400 years?
A) 7.41 grams
B) 0 grams
C) 6.16 grams
D) 4.91

17. Solve for *x*: $\log_2(x-1)=5$.

A) 33
B) 32
C) 31
D) 26

18. If $1500 is invested at 10 percent compounded continuously, what is the balance after 12 years?

A) $451.79
B) $1650
C) $4980.18
D) $4707.64

19. Find all the critical points of the function $f(x)=-2x^4+4x^2+1$.

A) 0, 1, -1
B) -1
C) $\frac{1}{2}$
D) none

20. Find the intervals of increase and decrease for $f(x)=\dfrac{10x-3}{-1x+10}$.

A) increasing on *x* < 10, decreasing on *x* > 10
B) increasing on *x* < 10 and *x* > 10
C) increasing on x £ 0.3 and on *x* > 10, decreasing on 0.3 < x £ 10
D) increasing on 0.3 < x £ 10 , decreasing on x £ 0.3 and on *x* > 10

21. Find the intervals of increase and decrease for the function $f(x)=x^2+3x-7$

A) decreasing for $x>\frac{-3}{2}$ and increasing for $x<\frac{-3}{2}$
B) decreasing for $x<\frac{-3}{2}$ and increasing for $x>\frac{-3}{2}$
C) decreasing for all *x*
D) increasing for all *x*

22. Determine where the graph of $f(x)=x^3-3x^2-9x+1$ is concave down.

A) *x* > 1
B) *x* < 1
C) *x* > -1
D) *x* < -1

23. A 5-year projection of population trends suggests that *t* years from now, the population of a certain community will be $P(t)=-t^3+9t^2+120t+55$ thousand.
1) At what time during the 5-year period will the population be growing most rapidly?
2) At what time during the 5-year period will the population be growing least rapidly?
3) At what time is the rate of population growth changing most rapidly?

A) *t* = 3 years; *t* = 0 years; *t* = 0 years
B) *t* = 0 years; *t* = 0 years; *t* = 4 years
C) *t* = 4 years; *t* = 3 year; *t* = 5 years
D) *t* = 4 years; *t* = 0 years; *t* = 4 years

24. Find the absolute maximum of the function $f(x)=x^5-x^4$ on the interval – 1 £ x £ 1 .

A) 0
B) 1
C) -1
D) -2

25. An apartment complex has 250 units. When the monthly rent for each unit is $400, all units are occupied. Experience indicates that for each $12 per month increase in rent, 4 units will become vacant. Each rented apartment costs the owner of the complex $40 per month to maintain. What monthly rent should be charges to maximize profit?

A) $119
B) $238
C) $357
D) $595

26. A commuter's train carries 600 passengers each day from a suburb to a city. It now costs $1 per person to ride the train. A study shows that 50 additional people will ride the train for each 5 cent reduction in fare. What fare should be charged in order to maximize total revenue?

A) 78 cents
B) 79 cents
C) 80 cents
D) 85 cents

27. Find the elasticity n of the demand function $q = \frac{3}{1+2p^2}$.

A) $n = -\frac{6}{1+2p^2}$

B) $n = -\frac{4p^2}{1+2p^2}$

C) $n = 4p$

D) $n = -4p^3$

28. A Florida citrus grower estimates that if 70 orange trees are planted, the average yield per tree will be 300 oranges. The average yield will decrease by 3 oranges per tree for each additional tree planted on the same acreage. How many trees should the grower plant to maximize the total yield?

A) 85 trees
B) 35 trees
C) 15 trees
D) 80 trees

29. The owner of a novelty store can obtain joy buzzers from the manufacturer for 50 cents apiece. He estimates he can sell 70 buzzers when he charges $1.4 apiece for them and that he will be able to sell 11 more buzzers for every 10 cent decrease in price. What price should he charge in order to maximize profit?

A) $1.17
B) $1.27
C) $1.07
D) $1.37

30. The derivative of $f(t) = \frac{1}{t^7}$ is

A) $\frac{-7}{t^8}$

B) $\frac{-7}{t^6}$

C) $\frac{7}{t^6}$

D) $\frac{1}{t^8}$

31. The graph of $f(x) = 5x^2 + 9$ has

A) a maximum at $x = -\frac{9}{10}$
B) a minimum at $x = -\frac{9}{10}$
C) a maximum at $x = 0$
D) a minimum at $x = 0$

32. Differentiate: $f(x) = \sqrt{x} + \frac{3}{\sqrt{x}}$

A) $\frac{1}{2\sqrt{x}} - \frac{3}{2\sqrt{x^3}}$

B) 0

C) 3

D) $\frac{1}{2\sqrt{x}} + \frac{3}{2\sqrt{x^3}}$

33. Find the second derivative of the given function and simplify your answer:

$f(t) = \frac{7}{4t+7}$

A) $f''(t) = \frac{224}{(4t+7)^3}$

B) $f''(t) = -\frac{224}{(4t+7)^3}$

C) $f''(t) = \frac{112}{4t+7}$

D) $f''(t) = -\frac{112}{4t+7}$

34. Evaluate the limit: $\lim_{x\to\infty} \frac{x^4 + 2x^2 - 3}{x^5}$

A) 1
B) -1
C) 0
D) ¥

35. Evaluate $\int \sqrt[3]{8x}\,dx$.

A) $\frac{8x}{3}+C$
B) $\frac{3x^{4/3}}{2}+C$
C) $\frac{3}{2x^{2/3}}+C$
D) $\frac{3x^{4/3}}{32}+C$

36. An object is moving so that its speed after *t* minutes is $5+2t+6t^2$ meters per minute. How far does the object travel between the end of minute 5 and the end of minute 6?

A) 198 meters
B) 798 meters
C) 188 meters
D) 688 meters

37. Evaluate $\int 2x^8-5x+6dx$

A) $\frac{2x^9}{9}-\frac{5x^2}{2}+6x+C$
B) $16x^7-5+C$
C) $2x^9 - 5x^2 + 6x + C$
D) $\frac{2x^9}{9}-\frac{5x^2}{2}+C$

38. In a certain section of the country, the price of chicken is currently \$2 per kilogram. It is estimated that *x* weeks from now the price will be increasing at a rate of $2\sqrt{x+1}$ cents per kilogram, per week. How much will chicken cost 8 weeks from now?

A) \$2.35
B) \$0.35
C) \$3.35
D) \$1.35

39. Use the fundamental theorem of calculus to find the area of the region under the line $y = 8x + 5$ above the interval 1 £ x £ 8 .

A) 305
B) 297
C) 287
D) 285

40. An animal population increases at the rate of $8x^3+3$ per year. What is the approximate increase in the animal population from the end of year 1 to the end of year 4?

A) 510
B) 519
C) 2049
D) 523

41. Records indicate that *t* hours past midnight, the temperature at the local airport was $f(t)=-0.2t^3+kt^2+5$ degrees Celsius. If the average temperature between 11 A.M. and 1 P.M. is 50 degrees C, what is *k*?

A) 1.90
B) 1.95
C) 2.05
D) 2.72

42. Money is transferred continuously into an account at the constant rate of \$1300 per year. The account earns interest at the annual rate of 7% compounded continuously. How much will be in the account at the end of 5 years?

A) \$2634.41
B) \$615000.25
C) \$26353.11
D) \$6500

43. It is estimated that *t* days from now a farmers crop will be increasing at the rate of $0.3t^2+0.6t+1$ bushels per day. By how much will the value of the crop increase during the next 2 days if the market price remains fixed at \$4 per bushel?

A) \$8
B) \$16
C) \$36
D) \$12

44. Money is transferred continuously into an account at the constant rate of $1000 per year. Assume the account earns interest at the annual rate of 7% compounded continuously. Compute the future value of the income stream over a 14 year period.

A) 1664.46
B) 47555.89
C) 95111.79
D) 23777.95

45. What is the domain of $f(x, y) = \dfrac{x}{(x^2+y^2-16)^{1/2}}$?

46. The monthly demand for product *A* is $D_1 = 50 - 10x + y$, while $D_2 = 20 + 3x - 5y$ is that for *B*. *A* sells for *x* dollars per item, *B* for *y* dollars per item. Express this total monthly revenue as a function of *x* and *y*.

47. Compute the indicated function value: if $f(s, t) = \dfrac{t}{s} + \dfrac{s}{t}$, then *f*(4, –1) =

A) –4.25
B) 3.75
C) –68.00
D) –3.75

48. Suppose *z* = *f*(*x*, *y*), $f_x(a, b) = 0$, $f_y(a, b) = 0$, $f_{xx}(a, b) = 5$, $f_{yy}(a, b) = 1$, and $f_{xy}(a, b) = 4$. Then (*a*, *b*) is

A) a relative minimum
B) a relative maximum
C) a saddle point
D) the test is inconclusive

49. Evaluate $\int (5x^3 - 3x + 4)\,dx$.

A) $\dfrac{5x^4}{4} - \dfrac{3x^2}{2} + 4x + C$
B) $15x^2 - 3 + C$
C) $5x^4 - 3x^2 + 4x + C$
D) $\dfrac{5x^4}{4} - \dfrac{3x^2}{2} + C$

50. Find the function whose tangent line has the slope e^x for each value of *x* and whose graph passes through (0, 2).

A) $e^x + 2$
B) $e^x + 1$
C) e^x
D) $e^x - 1$

51. An object is moving so that its velocity after *t* minutes is $v(t) = 5 + 8t + 9t^2$ meters per minute. How far does the object travel from the end of minute 4 to the end of minute 5?

A) 224 meters
B) 731 meters
C) –219 meters
D) –147 meters

52. $\int x^3\left(2x + 5 - 3\sqrt{x}\right)dx$

A) $\dfrac{x^4}{4}\left(x^2 + 5x - 2\sqrt{x^3}\right) + c$
B) $\dfrac{2x^5}{5} + 5x^3 - \dfrac{6}{5}\sqrt{x^5} + c$
C) $8x^3 + 15x^2 - \dfrac{21}{2}\sqrt{x^5} + c$
D) $\dfrac{2x^5}{5} + \dfrac{5x^4}{4} - \dfrac{2}{3}\sqrt{x^9} + c$

53. Evaluate $\int \dfrac{1}{4x}\,dx$

A) $\dfrac{\ln|x|}{4} + C$
B) ln | *x* | +C
C) $-\dfrac{2}{4x^2} + C$
D) $\dfrac{4}{(4x)^2} + C$

54. Evaluate $\int \left(\frac{1}{3}x-9\right)^{2/3} dx$

A) $\frac{x^2}{6}-9x+C$
B) $\frac{3}{5}\left(\frac{1}{3}x-9\right)^{5/3}+C$
C) $\frac{9}{5}\left(\frac{1}{3}x-9\right)^{5/3}+C$
D) $\left(\frac{1}{3}x-9\right)^{5/3}+C$

55. Evaluate the following integral: $\int 7e^{2x}\,dx$

A) $2.33e^{3x}+C$
B) $\frac{7}{2}e^{2x}+C$
C) $7e^{2x}+C$
D) $14e^{2x}+C$

56. Evaluate $\int_{-1}^{3}(3x-5)^4\,dx$. Express your answer as a decimal. Approximate to one decimal place.

A) 2,250.2
B) 2,251.6
C) 2,252.8
D) 2,253.4

57. Suppose the marginal cost is $C'(x)=e^{-0.1x}$, where *x* is measured in units of 300 items and the cost is measured in units of \$5,000. Find the cost corresponding to the production interval [300, 1,800] .

A) \$1,780
B) \$178
C) \$17,801
D) \$18,270

58. If $\int_a^b f(x)\,dx=-7$ and $\int_a^b g(x)\,dx=3$, find $\int_b^a [f(x)-g(x)]\,dx$.

A) 10
B) –21
C) –10
D) –4

59. The average value of $f(x)=x^3-3x+k$ over the interval 1 □ *x* □ 5 is 30. What is *k*?

A) 10
B) 8
C) 2
D) 0

60. Find the consumers surplus for a commodity whose demand function is $D(q)=30e^{-0.03q}$ dollars per unit if the market price is $p_0=\$21$ per unit. (Hint: Find the quantity q_0 that corresponds to the given price $p_0 = D(q_0)$.)

A) \$49.53
B) \$49.81
C) \$50.33
D) \$53.41

61. Money is transferred continuously into an account at the constant rate of \$1,100 per year. The account earns interest at the annual rate of 6% compounded continuously. How much will be in the account at the end of 2 years?

A) \$2,066.08
B) \$60,867.81
C) \$20,669.78
D) \$2,200

62. Money is transferred continuously into an account at the constant rate of \$1,100 per year. Assume the account earns interest at the annual rate of 4% compounded continuously. Compute the future value of the income stream over a 14 year period.

A) \$825.74
B) \$41,286.99
C) \$82,573.98
D) \$20,643.49

63. Demographic studies conducted in a certain city indicate that the fraction of the residents that will remain in the city for at least *t* years is $f(t) = e^{-t/50}$. The current population of the city is 500,000, and it is estimated that new residents will be arriving at the rate of 1,000 people per year. Assuming this estimate is correct, give an expression for the population as a function of *t*.

A) $50,000 + 500,000e^{-t/50}$
B) $40,500 + 500,000e^{-t/50}$
C) $40,500 + 450,000e^{-t/50}$
D) $50,000 + 450,000e^{-t/50}$

64. Compute f_x for $f(x,\ y) = 7xy^8$.

A) $7y^8$
B) $56xy^7$
C) $7x$
D) $56xy^7 + 7y^8$

65. Find $\frac{dz}{dt}$ if $z = 9x - 8y$, $x = t^3$, and $y = 11t$.

A) $\frac{dz}{dt} = 9t^3 - 88t$
B) $\frac{dz}{dt} = 27t^2 - 88$
C) $\frac{dz}{dt} = 3(9-8y)t^3 + 11(9x-8)$
D) $\frac{dz}{dt} = 9t^3 - 88$

66. A mall kiosk sells two different models of pagers, the Elite and the Diamond. Their monthly profit from pager sales is

$$P(x, y) = (x-40)(20-5x+6y)+(y-50)(30+3x-4y)$$

where *x* and *y* are the prices of the Elite and the Diamond respectively, in dollars. At the moment, the Elite sells for $32 and the Diamond sells for $40. Use calculus to estimate the change in monthly profit if the kiosk operator raises the price of the Elite to $33 and lowers the price of the Diamond to $38.

A) Profit will increase by about $26.
B) Profit will decrease by about $310.
C) Profit will increase by about $194.
D) Profit will stay the same.

67. Suppose $z = f(x, y)$, $f_x(a,\ b) = 0$, $f_y(a,\ b) = 0$, $f_{xx}(a,\ b) = -5$, $f_{yy}(a,\ b) = -2$, and $f_{xy}(a,\ b) = -3$. Then (*a*, *b*) is a

A) saddle point
B) a relative minimum
C) a relative maximum
D) the test (involving *D*) is inconclusive

68. The graph of the level curve $f(x, y) = C$ for $f(x,\ y) = x^2 - y$ and $C = -6$ is

A) a parabola
B) a circle
C) an ellipse
D) a straight line

69. A soft drink can is a cylinder *H* cm tall with radius *R* cm. Its volume is given by the formula $V = \pi R^2 H$. A particular can is 13 cm tall with radius 2 cm. Use calculus to estimate the change in volume that results if the radius is increased by 1 cm while the height remains at 13 cm.

A) The volume is increased by 78□ cm ^{3} .
B) The volume is increased by 52□ cm ^{3} .
C) The volume is increased by 4□ cm ^{3} .
D) The volume is increased by 26□ cm ^{3} .

70. Daily output $Q(K,\ L) = 10K^{1/3}L^{1/2}$ units. Use marginal analysis to estimate the change in daily output as a result of changing *L* from 625 to 626 while *K* remains constant at 216.

Answer Key

1. A
2. D
3. A
4. B
5. A
6. A
7. D
8. B
9. A
10. A
11. A
12. A
13. C
14. B
15. A
16. A
17. A
18. C
19. A
20. B
21. B
22. B
23. A
24. B
25. B
26. C
27. B
28. A
29. B
30. A
31. D
32. A
33. A
34. C
35. B
36. A
37. A
38. A
39. C
40. B
41. D
42. C
43. B
44. D

45. $x^2 + y^2 > 16$
46. $50x + 20y + 4xy - 10x^2 - 5y^2$
47. A
48. A
49. A
50. B
51. A
52. D
53. A
54. C
55. B
56. C
57. C
58. A
59. D
60. C
61. C
62. D
63. D
64. A
65. B
66. C
67. C
68. A
69. B
70. 1.2