Math165_ex2.3.5
A biologist models the effect of introducing a toxin to a bacterial colony by the function
P(t) = (t+1)/(t^2 + t + 4)
where P is the population of the coliny (in millions) t hours after the toxin is introduced.

(a) At what rate is the population changing when the toxin is introduced?
Is the population increasing or decreasing at this time?

(b) At what time does the population begin to increase?
By how much does the population decrease before it begins to decline?

```
> with(student);with(plots):
```

[D, *Diff*, *Doubleint*, *Int*, *Limit*, *Lineint*, *Product*, *Sum*, *Tripleint*, *changevar*, *completesquare*, *distance*, *equate*, *integrand*, *intercept*, *intparts*, *leftbox*, *leftsum*, *makeproc*, *middlebox*, *middlesum*, *midpoint*, *powsubs*, *rightbox*, *rightsum*, *showtangent*, *simpson*, *slope*, *summand*, *trapezoid*] (1)

```
> P:= proc(t)
    description `popultion in millions at t hours`;
    (t+1)/(t^2 + t + 4);
  end proc;
  `P(t)`:= P(t);
```

P := **proc**(*t*) **description** `*popultion in millions at t hours*`; (*t* + 1) / (*t*^2 + *t* + 4) **end proc**

$$P(t) := \frac{t+1}{t^2+t+4} \tag{2}$$

Find the derivative of P(t)

```
> P_prime:= proc(t)
    description `P'(t)`;
    diff(P(t),t);
  end proc;
  `P'(t)`:=P_prime(t);
  `P'(t)`:= simplify(%);
>
```

P_prime := **proc**(*t*) **description** `*P'(t)*`; *diff* (*P*(*t*), *t*) **end proc**

$$P'(t) := \frac{1}{t^2+t+4} - \frac{(t+1)\ (2\,t+1)}{\left(t^2+t+4\right)^2}$$

$$P'(t) := -\frac{t^2+2\,t-3}{\left(t^2+t+4\right)^2} \tag{3}$$

(a) At what rate is the population changing when the toxin is introduced?
Is the population increasing or decreasing at this time?

```
> `P'(0)`:=eval(P_prime(t),t=0);
```

$$P'(0) := \frac{3}{16} \quad (4)$$

Since the rate of change is positive at t = 0, the population is INCreasing when t = 0.

>

(b) At what time does the population begin to increase?
By how much does the population decrease before it begins to decline?

We seek t such that P'(t) = 0.
The problem can be restated:
Solve the equation $t^2 + 2\,t - 3 = 0$.
There are two solutions. Factor $t^2 + 2\,t - 3 = (t+3)(t-1)$.
The roots are t = 1 and t = 3. The "practical" solution is t = 1.
The total increase is P(1) - P(0).

```
> `Roots of P'(t) = 0`:=solve(P_prime(t) = 0,t);
  `Total Increase`:= P(1) - P(0);
```

$$\mathit{Roots\ of\ P'(t) = 0} := -3, 1$$

$$\mathit{Total\ Increase} := \frac{1}{12} \quad (5)$$

Getting Maple or another program to give the positive root is a little tricky.

```
>  solv_g:=proc(g,t)
   local soln, x:
   description `selects the positive solution   of g(x) = t,
   leaving x as a  variable.
   See   Maple Leaves Sec. 2.6`:
   solve({g(x)=t,x>0},x):
   assign(%):x:
   soln:=x;
  end proc;
```

solv_g := **proc**(*g*, *t*) (6)

local*soln*, *x*;

description

`selects the positive solution   of g(x) = t,
leaving x as a variable.
See   Maple Leaves Sec. 2.6`;

solve($\{g(x) = t, 0 < x\}$, *x*);

assign(`%`);

x;

soln := *x*

end proc

```
> t_max:= solv_g(P_prime,0);
```

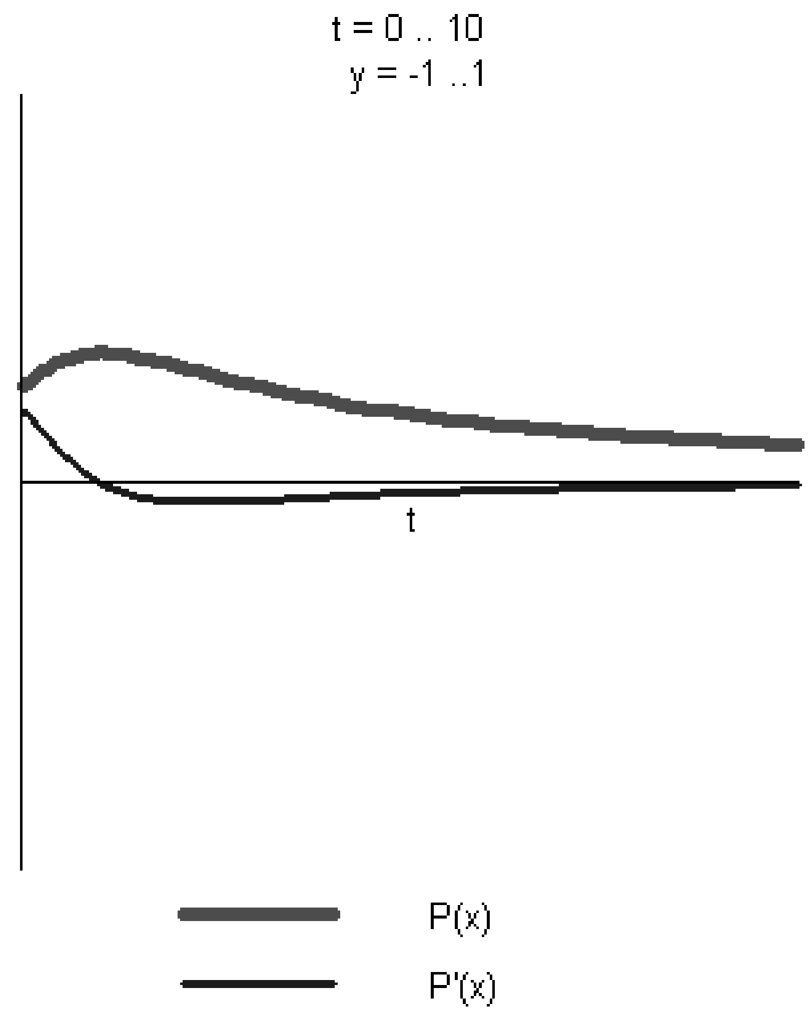
$$t\_max := 1 \quad (7)$$

```
> `Total change`:= P(t_max) - P(0);
```

$$Total\ change := \frac{1}{12} \quad (8)$$

Plots

```
> plotP_P_prime:=plot([P(t),P_prime(t)],t=0..10,P=-1.. 1, thickness
  = [3,2],labels=[`t`,``],legend=[`P(x)`,`P'(x)`],color=[red,blue],
  tickmarks=[0,0],
  title=`t = 0 .. 10\n y = -1 ..1`):
  display(plotP_P_prime);
```




```
>
```