math16509graphics.mw

Maple 10 Worksheet for Graphics - Calculus for Business.
N.B. These problems are a lot easier to do "by hand"
First load plots and student including student:-Calculus1

N.B. A Maple command such as eval(f(x),x=2) is the instruction
``Evaluate f(2)" or
``evaluate the function f(x) at x = 2."
a:= b assigns the value 'b' to the name 'a'
Use a semicolon ; to execute a 'Maple Command' and display the output.
Using a colon : executes a command without displaying the output.
I (JL) usually write functions as a procedure such as the "x square funtion:
square_function:= proc(x);x^2 ; end proc;
Other ways to write functions:
square_function:= x -> x^2;
`%` is the last computed expression (Simlar to ANS on your calculator).
"shading = zhue" gives the same color for equal values of z.

First define the functions on p.553:

$$f1(x,y) := -x\,y\,e^{-\frac{1}{2}x^2-\frac{1}{2}y^2} \quad (1)$$

$$f2(x,y) := -e^{-3\,x^2-3\,y^2} \quad (2)$$

$$f3(x,y) := x^4+y^4-2.3\,x^2-2.3\,y^2 \quad (3)$$

$$f4(x,y) := \left(0.8\,x^2+y^2\right)\,e^{1-1.4\,x^2-y^2} \quad (4)$$

$$f5(x,y,z) := x^2+y^2-0.2\,z^2 \quad (5)$$

$$f6(x,y,z) := -10\,x^2-10\,y^2+5\,z^2 \quad (6)$$

$$f7(x,y,z) := 3\,x^2+y^2+z^2 \quad (7)$$

Set up ranges

$$onerange := -1\,..1$$

$$tworange := -2\,..2$$

$$threerange := -3\,..3$$

$$fourrange := -4\,..4$$

$$fiverange := -5\,..5 \quad (8)$$

Some other functions
Graph is a cone with vertex at (0,0,0)

$$fcone(x,y) := \sqrt{x^2+y^2} \quad (9)$$

A paraboloid -cross section with planes parallel to z -axis are parabolas
Cross sections with planes parallel with z axes are ellipses(circles).

$$fpara(x,y) := x^2+y^2 \quad (10)$$

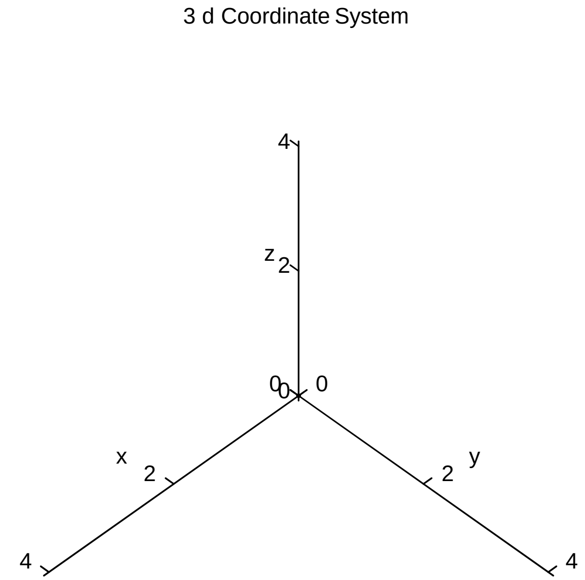
An ellipsoid (upper half) - cross sections with planes are ellpses(or circles)

(11)

$$fellips(x,y) := \sqrt{9-3\,x^2-2\,y^2} \quad (11)$$

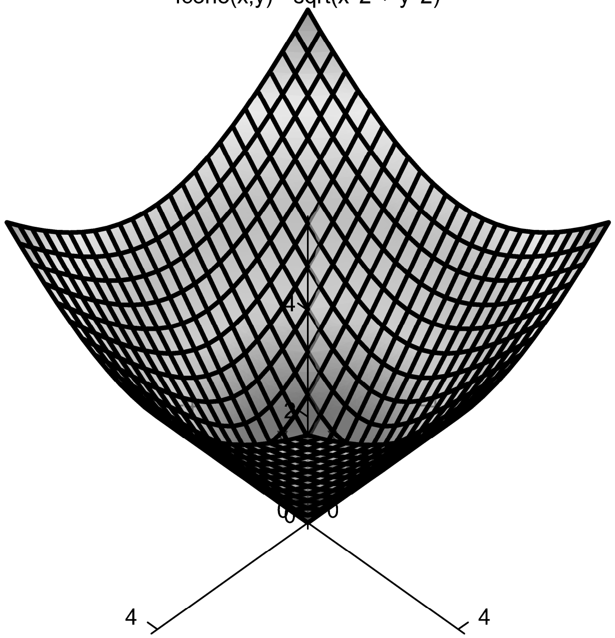
Ways to think of graphs.

All "full" graphs are on a 3 dimensional coordinte system. Show 0 \leq x \leq 4, 0 \leq y \leq 4, 0 \leq z \leq 4,



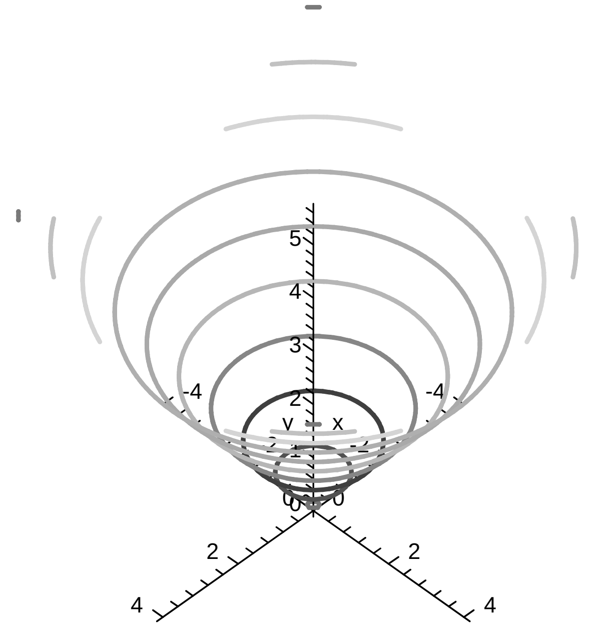
Now plot the "full"picture of a cone.

fcone(x,y) =sqrt(x^2 + y^2)

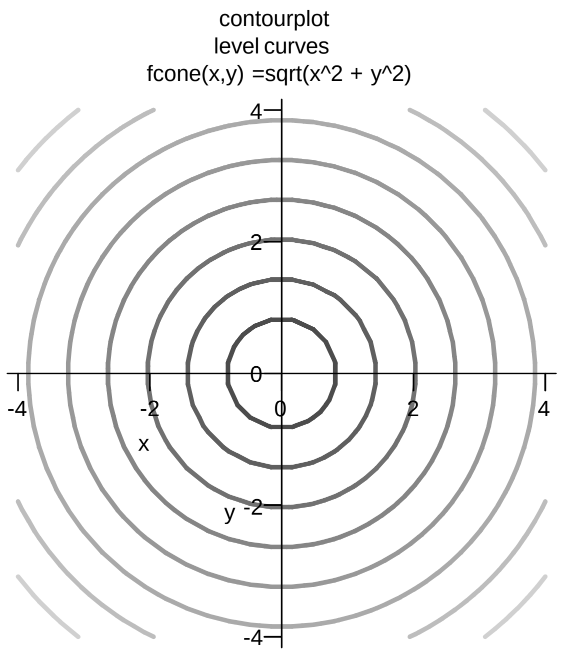


Another way to unsderstand the graph is to see what the curves f(x,y) = constant appear.This is called a Contour Plot.3d"

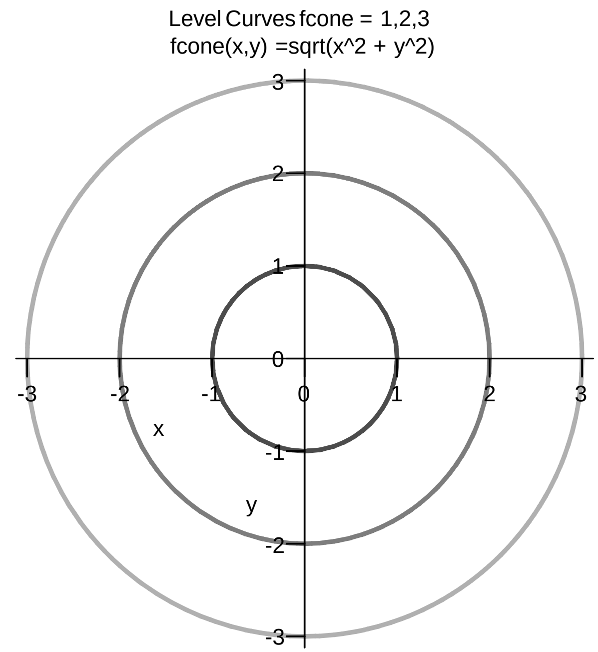
fcone(x,y) =sqrt(x^2 + y^2)



Next (and most useful!) is to look at the curves f(x,y) = constant as as cirves in the x-y plane. These are called level curves of the function. In this case the level corves are circles.
The plot is sometimes called a "Countour Plot"or "Level Curve Plot"



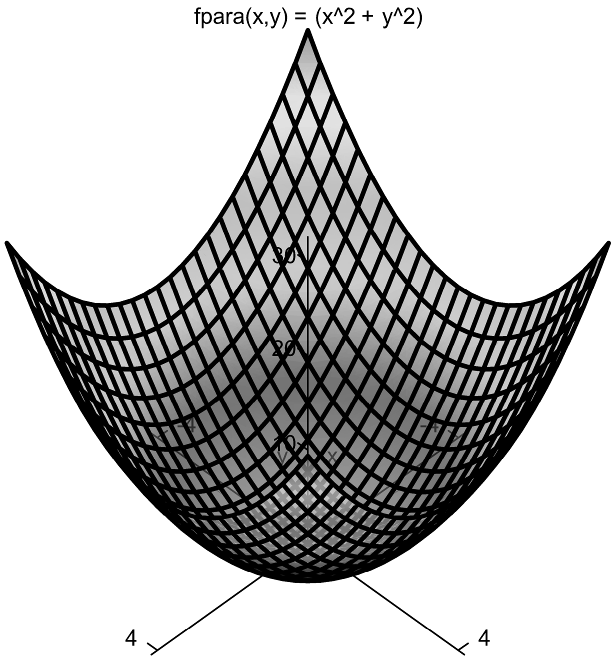
We can even specify the "constant values" of f(x,y),e.g. f(x,y) = sqrt of 1,2,or 3.



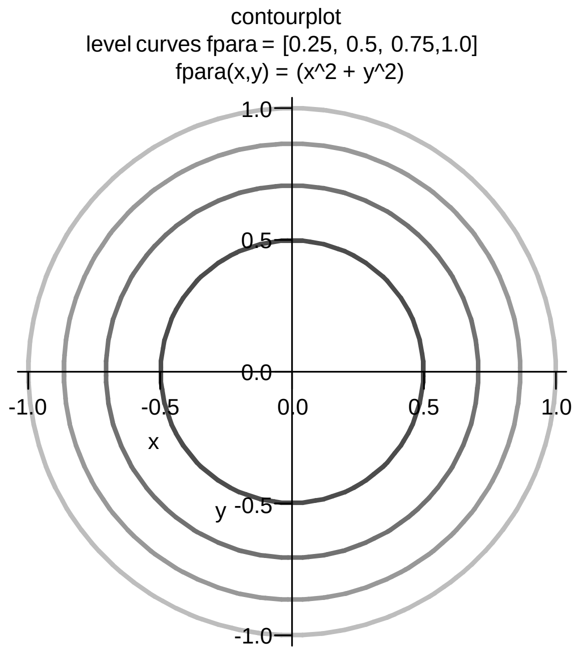
Now consider the paraboloid

$$fpara(x,y) := x^2 + y^2 \quad (12)$$

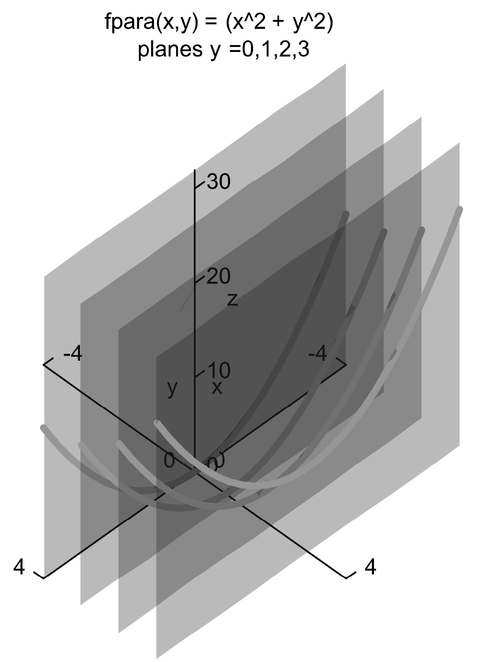
The "full graph" in 3 dimensions



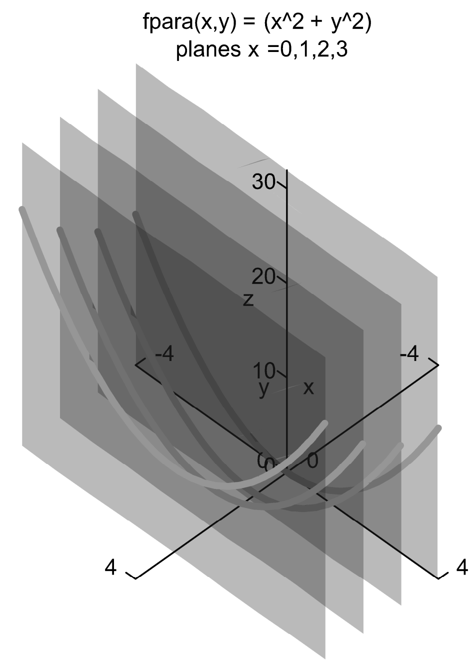
The "countour plot 3d"
The Level Curves: f = 0.25, 0.5, 0.75,1.0



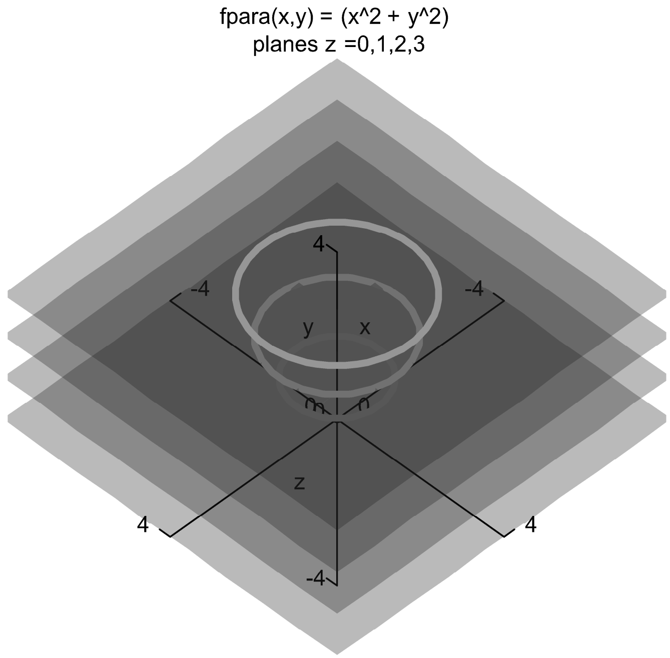
Another way to understand the graph is to to freeze one of the variables, e.g. y, and consider the function **of x.**
The function of x is f_y(x) = x^2 + y^2 which is a parabola opening upward in the "x-z plane" with vertex at
(x=0,z= y^2).
The Idea is Exploited in taking "partial derivative wrt x"



Next we "freeze x" and consider the function of x defined by fy(x) = x^2 + y^2 graphed in a "y-z plane"

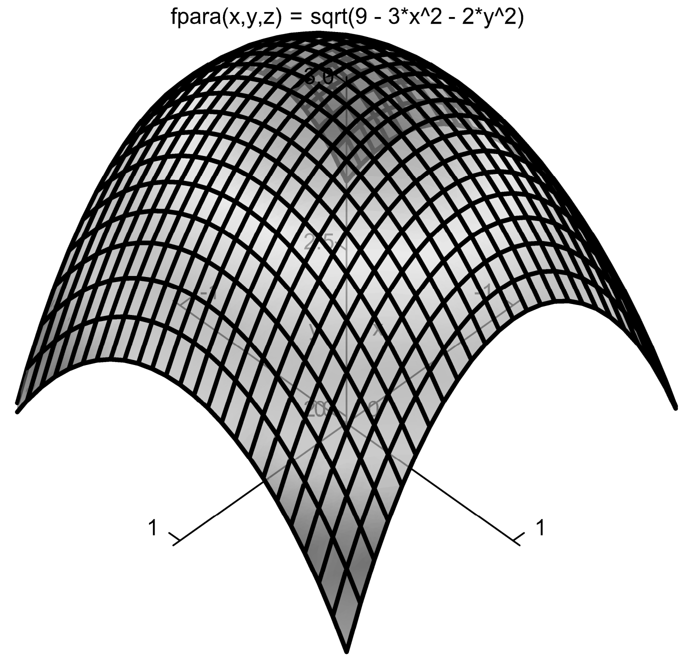


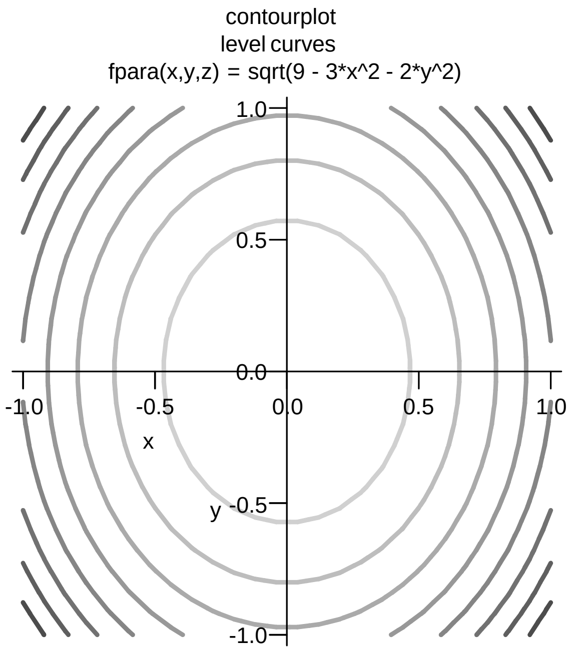
Looking at planes "z = constant" brings us back to the "Contour Plot 3d" or "Level Curves"



Plots for the (half) ellipsoid

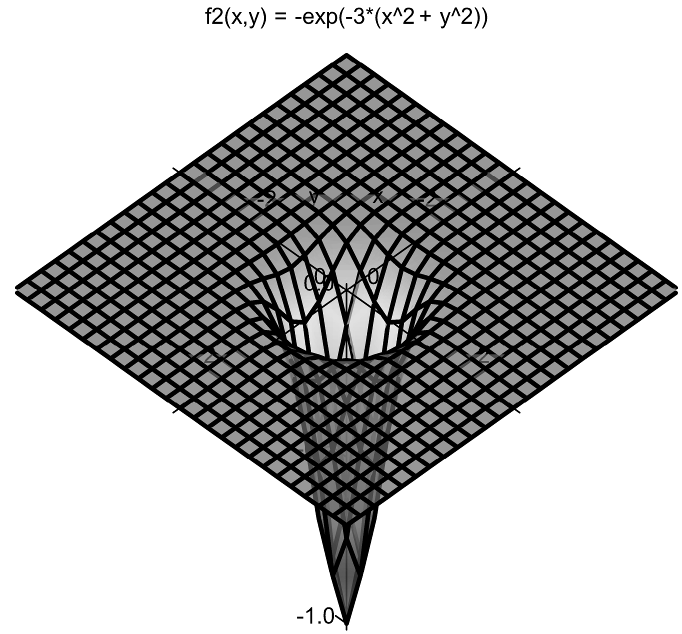
$$fellips(x,y) := \sqrt{9-3\,x^2-2\,y^2}$$

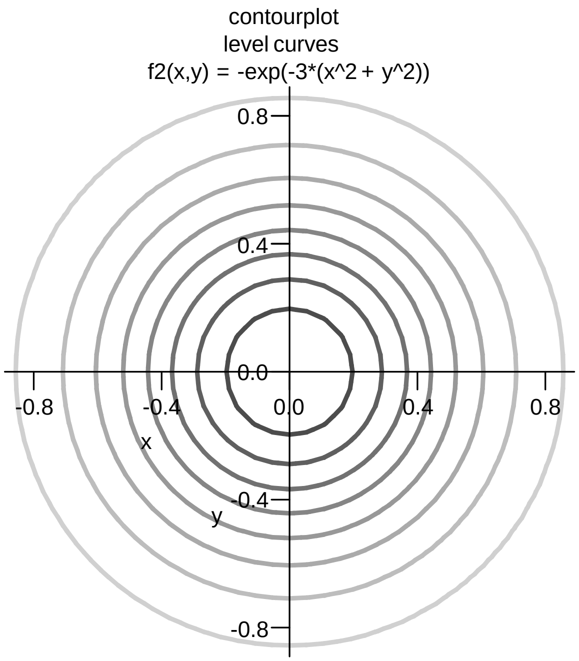


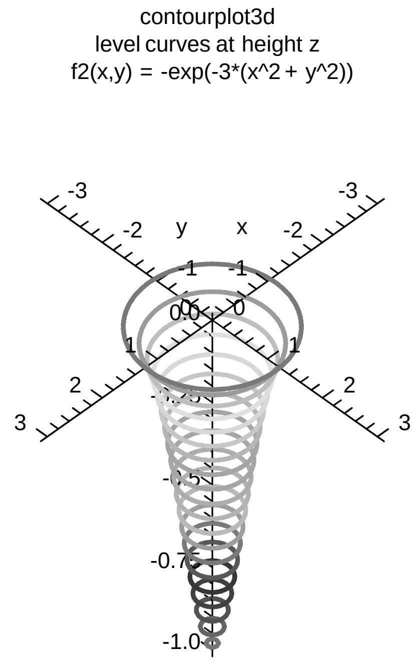


Another function which depends only on (x^2 +y^2) and which is very small away from he origin.

$$f2(x,y) := -e^{-3x^2-3y^2}$$



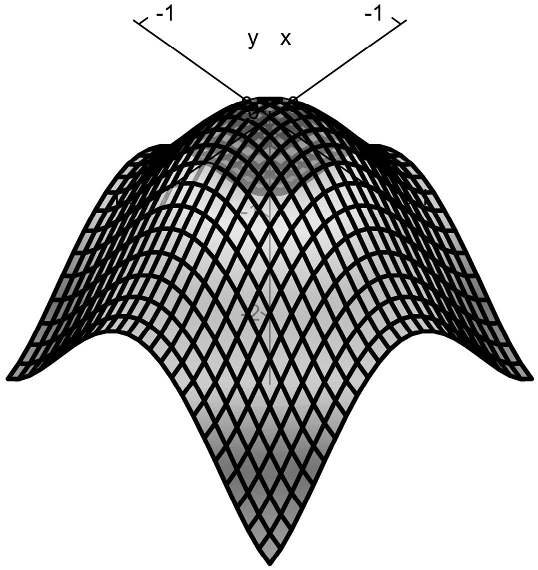




For the next function, f3, the function is negative for |x^2 + y^2| near 0 and positive for |x^2 + y^2| "large".

$$f3(x,y) := x^4 + y^4 - 2.3\,x^2 - 2.3\,y^2$$

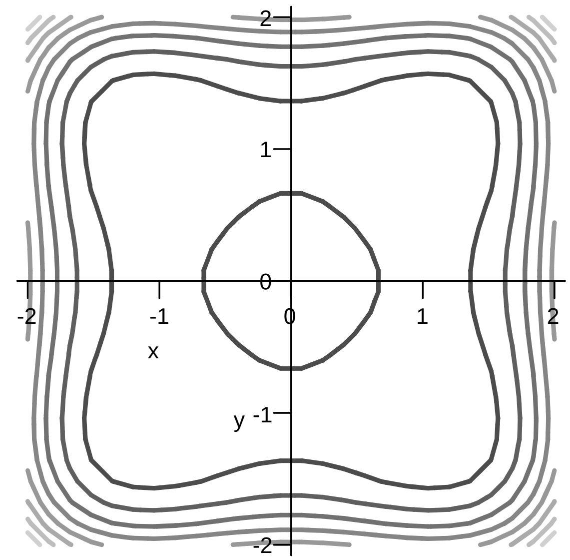
f3(x,y) = x^4 + y^4 - 2.3*(x^2 + y^2)



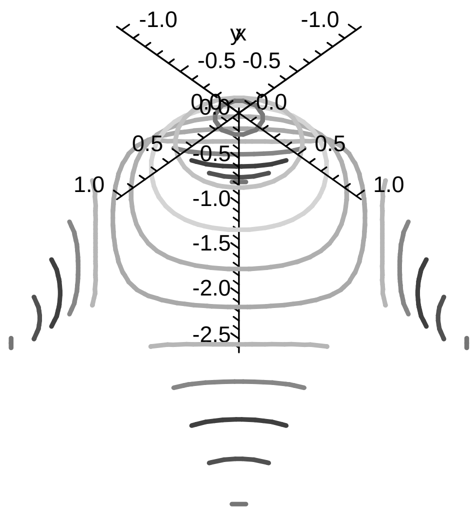
contourplot

level curves z =[0.25,0.50,0.75,1.0]

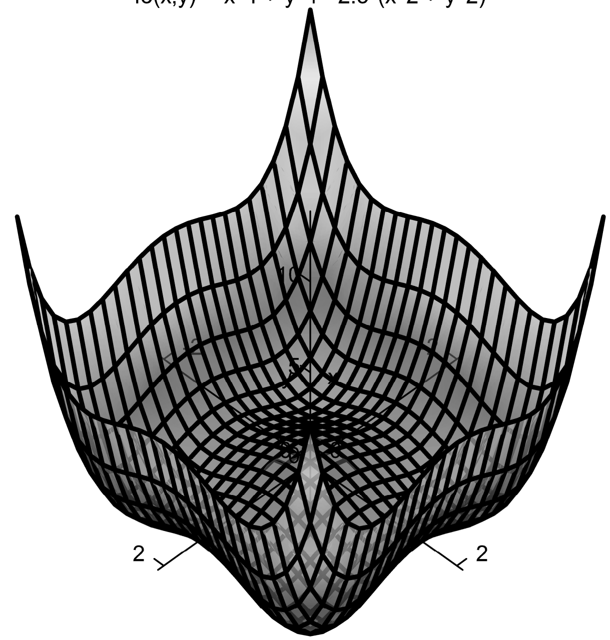
f3(x,y) = x^4 + y^4 - 2.3*(x^2 + y^2)

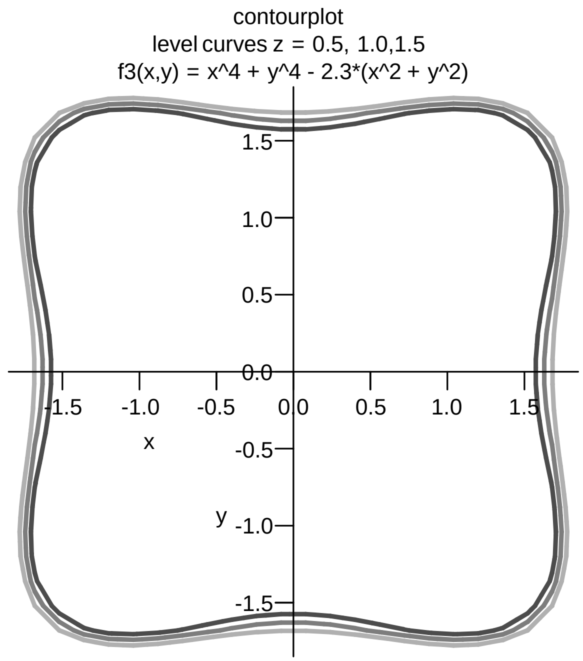


contourplot3d
level curves at height z
f3(x,y) = x^4 + y^4 - 2.3*(x^2 + y^2)



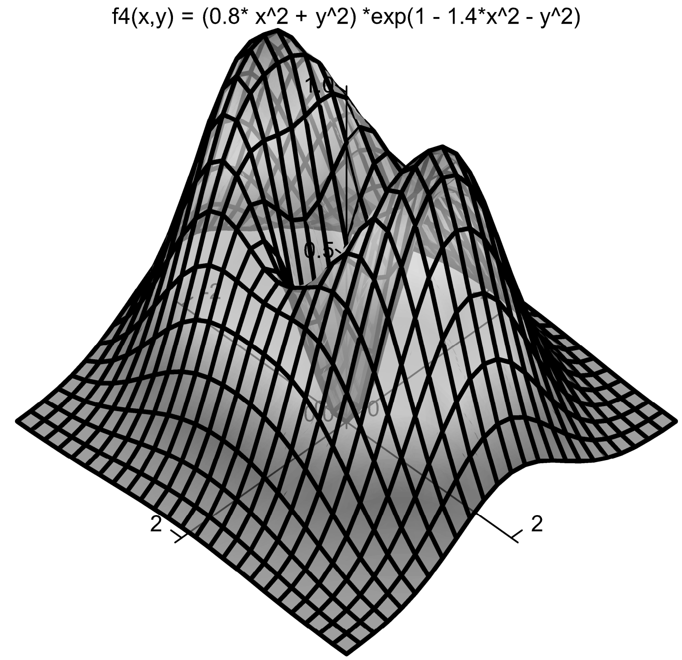
f3(x,y) = x^4 + y^4 - 2.3*(x^2 + y^2)

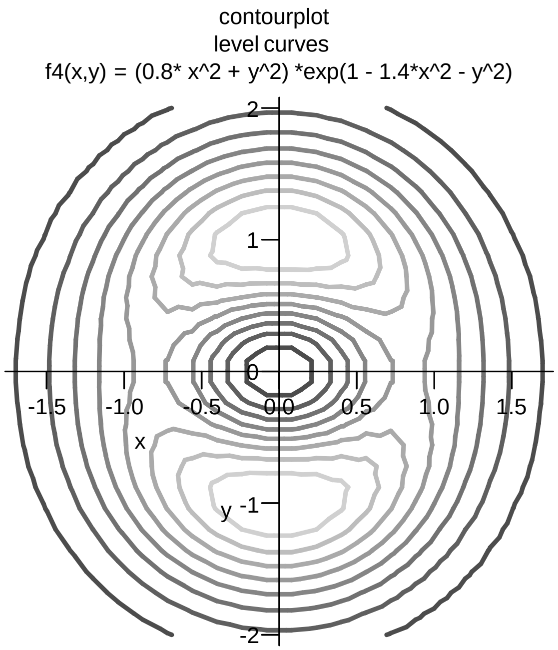




Two peaks and a valley

$$f4(x,y) := \left(0.8\,x^2 + y^2\right)\,e^{1 - 1.4\,x^2 - y^2}$$

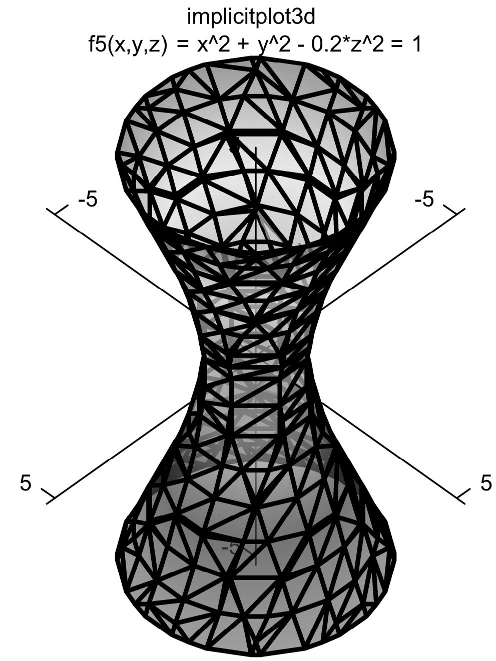




[An omitted example]

Implicit Plots of a Surface f5(x,y,z) = 1.."Hyperboloid of One Sheet'

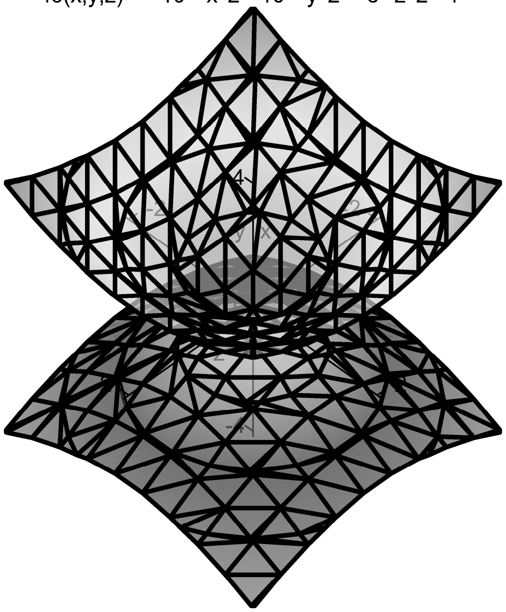
$$f5(x,y,z) := x^2 + y^2 - 0.2\, z^2$$



Another "Hyperboloid of Two Sheets"

$$f6(x,y,z) := -10\, x^2 - 10\, y^2 + 5\, z^2$$

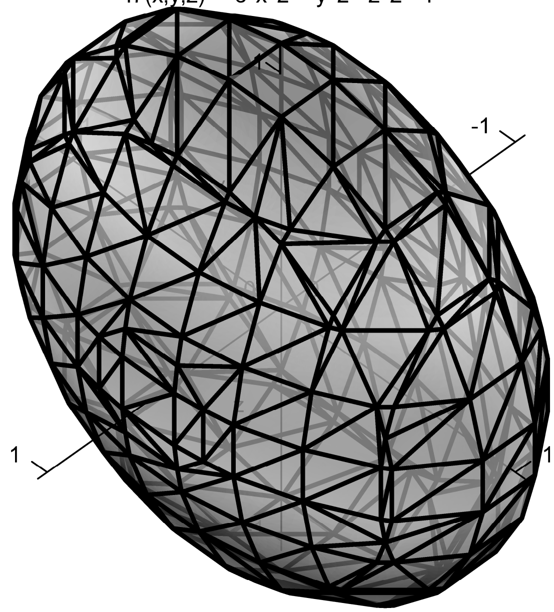
implicitplot3d
f6(x,y,z) = -10 * x^2 - 10 * y^2 + 5 *z^2 =1



The Full Ellipsoid

$$f7(x,y,z) := 3\,x^2 + y^2 + z^2$$

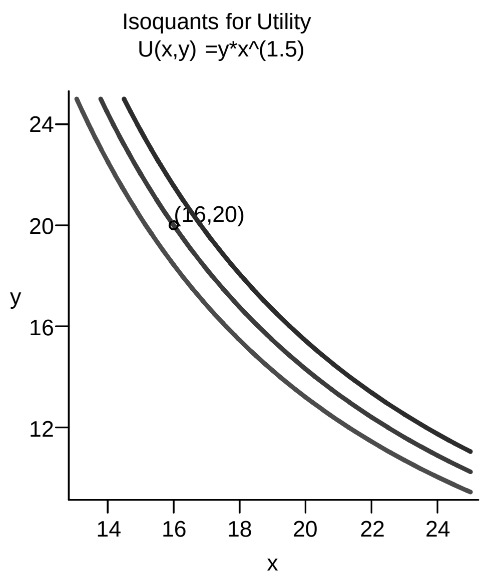
implicitplot3d
f7(x,y,z) = 3*x^2 + y^2 +z^2 =1



Example 7.1.9
Level Curves for a Utility Function

$$U(x,y) := x^{1.5}\,y$$

$$U\_0 := 1280.000000$$


Isoquants for Utility
U(x,y) =y*x^(1.5)
(16,20)
24
20
16
12
y
14
16
18
20
22
24
x