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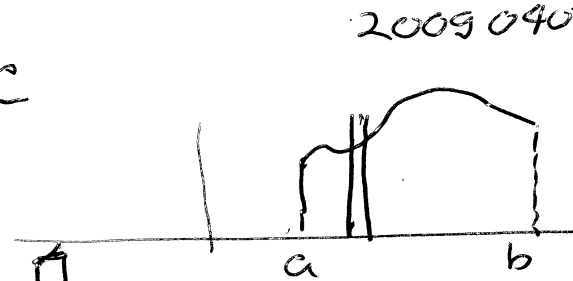
$$\int f(x)\,dx = F(x) + C$$

"Area" Problem under a graph
above x-axis $a \le x \le b$



Simple Model : Rectan
Small slice

f1 $\approx f(x)\,\Delta x$ ($\pm$ a small err)

base Δx

Now the process is to take "small slices", add

$$\text{Area} = \underset{\substack{\text{Sum} \\ \text{(sigma}}}{\sum} f(x)\,\Delta x$$; x moves from a to

by steps Δx (pos

$$\sum_{x = a..b \text{ by } \Delta x} f(x)\,\Delta x$$
"from a to b"

Riemann Sum, and then take

$$\lim_{\Delta x \to 0} \sum f(x)\,dx = \text{"integral of } f(x) \text{ "} dx\text{"}$$

from a to b

(Actually calculating - computers are very good)
and there is a "closed form" related to anti D

FTC (p. 391)

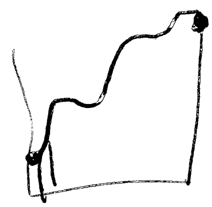
If the function F(x) is an anti D of f(x) on [a,b]

then $$\int_a^b f(x)\,dx = F(b) - F(a)$$

Notation $F(x)\Big|_a^b$ or $F(x)\Big|_{x=a}^{x=b}$

Example (1°) Area under f [pos] above [a,b]

$$\int_a^b f(x)\,dx = F(b)-F(a)$$



(2°) Change in F

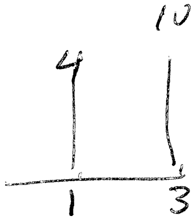
$$F(b)-F(a) = \sum_{\text{small changes}} \Delta F$$

$$\approx \sum_{x \text{ from } a..b} F'(x)\,\Delta x \xrightarrow[\Delta x \to 0]{} \int_a^b F'(x)\,dx$$

Area of trapezoid under $y=3x+1$, above $1 \le x \le 3$

— Integ $\displaystyle\int_1^3 3x+1\,dx = 3\frac{x^2}{2}+x\Big|_{x=1}^{x=3}$

$$= \left(\frac{27}{2}+3\right)-\left(\frac{3}{2}+1\right) = 14$$



— Geometry $\underset{\text{width}}{(3-1)}\left(\frac{(3\cdot3+1)-(3\cdot1+1)}{2}\right) = 14$

$2\left(\frac{10+4}{2}\right) = 14$

Numerical Sum $Y1 = 3*X+1$ optional on TI-83

fnInt(Y1 (X), X, 1, 3) = 14

MATH 9 Var-Yvar-E1

RULES $\displaystyle\int_a^b Af(x) \pm B\,g(x)\,dx = A\int_a^b f(x)\,dx \pm B\int_a^b g(x)\,dx$

NEW $\displaystyle\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$

(f(x) good on interval containing all 3: a, b, c

Application ~~the~~ 20090403 3/3

Example (5.3.9): $MC = \frac{dC}{dx} = 3(q-4)^2$

increase in cost boosting from 6, to 10

$$\int_6^{10} 3(q-4)^2\,dq = (q-4)^3 \Big|_{q=6}^{q=10}$$

SUBS $\int 3(q-4)^2\,dq \underset{\substack{u=q-4\\ du=dq}}{=} \int 3u^2\,du = 3\frac{u^3}{3} = (q-4)^3 + C$

$$\int_6^{10} 3(q-4)^2\,dq = \int_{\boxed{q}}^{\boxed{q=}} 3u^2\,du = u^3 \Big|_{q=}^{q=} \quad \text{MIXED}$$

NOT u!

Income Stream. 165stream.pdf