**Math 165: Income and Investment Streams**

## Simple Model

If a *present value* $P$ is invested at time $t = 0$ with continuous compounding (CC) at rate $r$, the *future value* at time $t = T$ is

$$B = B(T) = Pe^{rT} = P(0)e^{rT}.$$

If we wish to have *future value* $B$ at time $t = T$, we should invest a *present value* $P$ given by

$$P = P(0) = Be^{-rT} = B(T)e^{-rT}.$$

If we wish to withdraw amounts $B_1, B_2, \ldots, B_N$, at times $T_1, T_2, \ldots, T_N$, we must have present value

$$P = P_1 + P_2 + \ldots + P_N = B_1e^{-rT_1} + B_2e^{-rT_2} + \ldots + B_Ne^{-rT_N} = \sum B_ie^{-rT_i}.$$

## Continuous Model – Income Stream

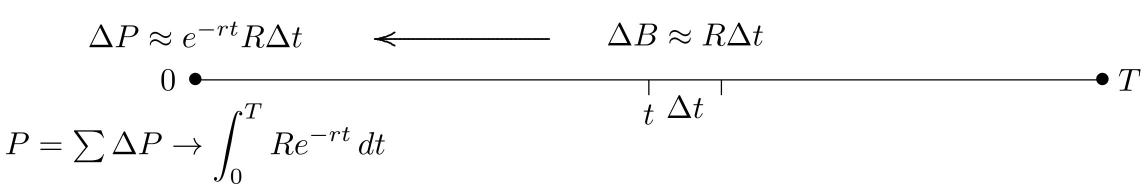
We wish to withdraw a *continuous income stream* – to withdraw continuously at a rate $R$ [dollars/year] for $T$ [years].

At a typical time $t$, over a period $\Delta t$, we will withdraw $\approx R\Delta t = \Delta B$, a future value at time $t$. Thus we need a present value (investment) $\Delta P \approx e^{-rt}R\Delta t$. The total present value needed is

$$P = \sum \Delta P \approx \sum_{t \text{ from } 0 \text{ to } T} e^{-rt}R\Delta t \approx \int_0^T Re^{-rt}\, dt.$$

Figure 1.

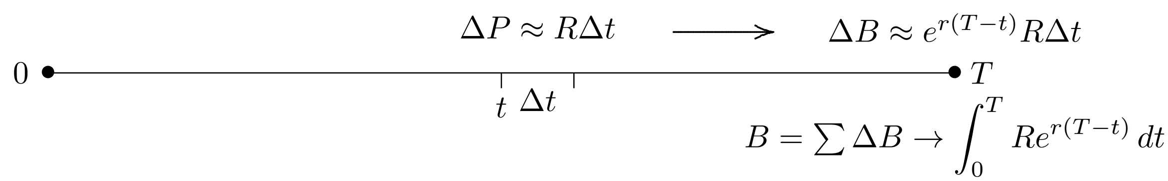
Present Value $P$ of an Income Stream $R$, $0 \le t \le T$



Similarly, if we *invest* continuously at rate $R$ [dollars/year] for $T$ [years], the *future value* $B$ at time $T$ will be given by

$$B = \int_0^T R\, e^{r(T-t)}\, dt.$$

Figure 2.
Future Value $B$ of an Investment Stream $R$, $0 \le t \le T$




**Inflation Adjustments**

Similar arguments can be made if the rate, $R = R(t)$, depends on $t$. For example, the rate of contribution (investment) or income (revenue) might be continuously adjusted for inflation. In this case the formulas become:

- If we wish to withdraw a *continuous income stream* – to withdraw continuously at a rate $R(t)$ [dollars/year] for $T$ [years], we need a present value

$$P = \int_0^T R(t)\, e^{-rt}\, dt.$$

In particular if we make a *cost of living adjustment* (COLA), of $r_1\%$ annually,

$$R(t) = R_0 e^{r_1 t},$$
$$P = \int_0^T R_0\, e^{(r_1 - r)t}\, dt$$

- If we *invest* continuously at rate $R(t)$ [dollars/year] for $T$ [years], the *future value* $B$ at time $T$ will be given by

$$B = \int_0^T R(t)\, e^{r(T-t)}\, dt.$$