10.6, #6. $f(x)$ is an odd function on $[-\pi, \pi]$. So $f(x)\cos nx$ and $f(x)\sin nx$ are respectively odd and even functions on $[-\pi, \pi]$. In particular,

$$\begin{aligned} a_1 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos x\,dx = 0 \\ a_2 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos 2x\,dx = 0 \\ a_3 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos 3x\,dx = 0 \end{aligned}$$

and

$$\begin{aligned} b_1 &= \frac{2}{\pi}\int_{0}^{\pi} \sin x\,dx = \frac{4}{\pi} \\ b_2 &= \frac{2}{\pi}\int_{0}^{\pi} \sin 2x\,dx = 0 \\ b_3 &= \frac{2}{\pi}\int_{0}^{\pi} \sin 3x\,dx = \frac{4}{3\pi} \end{aligned}$$

Moreover, $a_0 = 0$ since a_0 is the average value of $f(x)$ on $[-\pi, \pi]$. Thus $F_1(x) = F_2(x) = (4/\pi)\sin x$, $F_3(x) = (4/\pi)\sin x + (4/3\pi)\sin 3x$.

10.6, #7. $f(x)$ is an even function on $[-\pi, \pi]$. So $f(x)\sin nx$ and $f(x)\cos nx$ are respectively odd and even functions on $[-\pi, \pi]$. In particular,

$$\begin{aligned} b_1 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin x\,dx = 0 \\ b_2 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin 2x\,dx = 0 \\ b_3 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin 3x\,dx = 0 \end{aligned}$$

and

$$
\begin{aligned}
a_1 &= \frac{2}{\pi}\int_0^\pi x\cos x\,dx = \frac{2}{\pi}(x\sin x + \cos x)\Big|_0^\pi = -\frac{4}{\pi} \\
a_2 &= \frac{2}{\pi}\int_0^\pi x\cos 2x\,dx = \frac{2}{\pi}((x\sin 2x)/2 + (\cos 2x)/4)\Big|_0^\pi = 0 \\
a_3 &= \frac{2}{\pi}\int_0^\pi x\cos 3x\,dx \;= \frac{2}{\pi}((x\sin 3x)/3 + (\cos 3x)/9)\Big|_0^\pi = -\frac{4}{9\pi}
\end{aligned}
$$

Finally, $a_0 = \pi/2$ since a_0 is the average value of $f(x)$ on $[-\pi,\pi]$. Thus $F_1(x) = F_2(x) = (\pi/2) - (4/\pi)\cos x$, $F_3(x) = (\pi/2) - (4/\pi)\cos x - (4/9\pi)\cos 3x$.

10.6, #9. $f(x) = x^2$ is an even function on $[-\pi,\pi]$. So $f(x)\sin nx$ and $f(x)\cos nx$ are respectively odd and even functions on $[-\pi,\pi]$. In particular,

$$
b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = 0
$$

and

$$
\begin{aligned}
a_n &= \frac{2}{\pi}\int_0^\pi x^2\cos nx\,dx \\
&= \frac{2}{\pi}\left(\frac{x^2\sin nx}{n} + \frac{2x\cos nx}{n^2} - \frac{2\sin nx}{n^3}\right)\Big|_0^\pi \\
&= 4/n^2\cos n\pi = (-1)^n 4/n^2
\end{aligned}
$$

Finally, $a_0 = \pi^2/3$ since a_0 is the average value of $f(x)$ on $[-\pi,\pi]$. Thus

$$
\begin{aligned}
&F_1(x) = \pi^2/3 - (4/\pi)\cos x, \\
&F_2(x) = \pi^2/3 - (4/\pi)\cos x + (1/\pi)\cos 2x, \\
&F_3(x) = \pi^2/3 - (4/\pi)\cos x + (1/\pi)\cos 2x - (4/9\pi)\cos 3x.
\end{aligned}
$$

10.6, #15. $g(x) = 1$ for $-\pi/2 \le x \le \pi/2$ and $= 0$ elsewhere. Therefore g is even with average value $1/2$, so $a_0 = 1/2$, $b_k = 0$, and

$$a_k = \frac{2}{\pi}\int_0^{\pi/2} \cos(kx)\,dx = \begin{cases} 0, & k \text{ even}, \\ +\frac{2}{k\pi}, & k = 1, 5, 9 \cdots \\ -\frac{2}{k\pi}, & k = 3, 7, 11, \cdots \end{cases}.$$

The series is

$$\frac{1}{2} + \frac{2}{\pi}\Big(\cos x - \frac{1}{3}\cos(3x) + \frac{1}{5}\cos(5x) - + \cdots\Big).$$

This square-wave is shifted by $\pi/2$ to the left compared to the square-wave of example #1, $g(x) = f(x + \pi/2)$. Replacing x by $x + \pi/2$ in the Fourier series for f gives a series whose general term is

$$\frac{2}{k\pi}\sin(k(x + \pi/2)) = \begin{cases} +\frac{2}{k\pi}\cos(kx), & \text{for } k = 1, 5, \cdots \\ -\frac{2}{k\pi}\cos(kx) & \text{for } k = 32, 7, \cdots \end{cases}.$$

10.6, #19. For the flute the first and second harmonics are equally dominant, while higher harmonics are weak. For the basson the dominant harmonic is the third; the second harmonic is moderate, and all other harmonics including the first harmonic are weak.

10.6, #20.