10.5, #10. $P_n(1)$ approximates $\cos 1$ to four decimal places if the corresponding error $E_n$ satisfies $|E_n| < 0.00005$. By the preceding problem we want $1/(n+1)! < 0.00005$. This happens for $n = 7$ since $8! = 40320$ and $1/8! = 0.000025$. For 6 decimal places, we want $1/(n+1)! < 0.0000005$. This happens for $n = 9$.

10.5, #12. The higher derivatives of $f(x) = \cos x$ are $\pm \sin x$ and $\pm \cos x$. So $|f^{n+1}(x)| \leq 1$ for all $x$ and we may take $M = 1$ in the boxed remark on page 631. Then

$$|E_n| \leq \frac{1}{(n+1)!}|x|^{n+1}.$$

But as noted on page 633 of the text, the ratio $x^{n+1}/(n+1)! \to 0$ as $n \to \infty$. So the Taylor series for $\cos x$ converges to $\cos x$.

10.5, #14. For $-0.1 \leq x \leq 0.1$, $E_1$ lies between the $x$-axis and the graph of $x^2$ and looks like $\frac{1}{2}x^2$.
For $-0.1 \leq x \leq 0.1$, $E_2$ lies between the $x$-axis and the graph of $x^3$. A good range for $y$ is $-0.001 \leq y \leq 0.001$.

10.6, #1. This is a Fourier series.

10.6, #2. This is not a Fourier series.

10.6, #3. This is not a Fourier series. Although $\sin(x+n) = (\cos n)\sin x + (\sin n)\cos x$ is a combination of $\sin x$ and $\cos x$ that could appear in a Fourier series, the fact that $\sin(x+n)$ occurs for $n = 1, 2, 3, \ldots$ means that such a Fourier series would have

$$1 + \cos 1 + \cos 2 + \cos 3 + \cos 4 + \cdots$$

as the coefficient of $\sin x$. This sum does not converge.

10.6, #5 The next Fourier approximation would be

$$\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \frac{\sin 7x}{7}.$$

The square wave function has period $2\pi$ and has the form

$$f(x) = \begin{cases} -h & -\pi \leq x < 0 \\ h & 0 \leq x < \pi. \end{cases}$$

Since the coefficient of $\sin x$ in our series is 1 we must have

$$1 = b_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin x\,dx = \frac{2}{\pi}\int_0^{\pi} h\sin x\,dx = \frac{4h}{\pi}.$$

Hence $h = \pi/4$. $f(x)$ is not continuous at $k * \pi$ for any integer $k$.