

10.4, #15.

$$\begin{aligned} 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \cdots + \frac{3}{2^{10}} \\ &= 3 \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^{10}} \right) \\ &= \frac{3(1 - 1/2^{11})}{1 - 1/2} \\ &= \frac{3(2^{11} - 1)}{2^{10}}. \end{aligned}$$

10.4, #23. (a) The height to which the ball rises after it hits the floor for the n -th time is $10(3/4)^n$ feet.

(b) The ball has traveled a total vertical distance of 10 feet when it hits the floor the first time, $10 + 2(10)(3/4) = 25$ feet the second time, $10 + 2(10)(3/4) + 2(10)(3/4)^2 = 36.25$ feet the third time, and $10 + 2(10)(3/4) + 2(10)(3/4)^2 + 2(10)(3/4)^3 = 44.69$ feet the fourth time.

(c) When the ball hits the floor the n -th time, it has traveled a total vertical distance

$$\begin{aligned} 10 + 2(10)(3/4) + 2(10)(3/4)^2 + 2(10)(3/4)^3 + \cdots + 2(10)(3/4)^{n-1} \\ &= 10 + 15[1 + (3/4) + (3/4)^2 + \cdots + (3/4)^{n-2}] \\ &= 10 + 15 \left(\frac{1 - (3/4)^{n-1}}{1 - (3/4)} \right) \\ &= 10 + 60(1 - (3/4)^{n-1}) \text{ feet.} \end{aligned}$$

10.4, #30. The present value of a \$1000 bond is

$$1000(1.06)^{-10} + 50[(1.06)^{-1} + (1.06)^{-2} + \cdots + (1.06)^{-10}]$$

The term $1000(1.06)^{-10}$ appears since the principal will have been held by the issuer of the bond for 10 years. The term $50(1.06)^{-j}$ is the present value of the coupon paid at the end of the j -th year. Thus present value of the bond is

$$1000(1.06)^{-10} + \frac{50}{1.06} \left(\frac{1 - (1.06)^{-10}}{1 - (1.06)^{-1}} \right) = 926.40 \text{ dollars.}$$

10.5, #1–2. The derivatives of $f(t) = e^t$ are e^t . Thus $|f^n(t)| \leq e^{0.5}$ on $[0, 0.5]$. In particular, $E_0 = f(t) - P_0(t)$ and $E_2 = f(t) - P_2(t)$ satisfy the inequalities

$$|E_0| \leq e^{0.5}(0.5) = 0.8243, \quad |E_2| \leq e^{0.5}(0.5)^3/3! = 0.0343$$

by the second boxed remark on page 631. $P_0(t)$ and $P_2(t)$ are underestimates for e^t since they are truncations of the series $1 + t + t^2/2! + t^3/3! + \cdots$, which contains only non-negative numbers when t is in $[0, .05]$.

10.5, #9. The $(n + 1)$ -st derivative of $f(x) = \cos x$ is $-\sin x$, $-\cos x$, $\sin x$, or $\cos x$ according as $n = 4t$, $4t + 1$, $4t + 2$, or $4t + 3$ for some non-negative integer t . In particular,

$$|f^{(n+1)}(x)| = \begin{cases} |\sin x| & \text{if } n \text{ is even} \\ |\cos x| & \text{if } n \text{ is odd} \end{cases}$$

If $0 \leq x \leq 1$, then $|f^{(n+1)}(x)| \leq \sin 1 = 0.841$ for even n , and $|f^{(n+1)}(x)| \leq \cos 0 = 1$ for odd n . The error term is thus bounded above by $(0.841/(n + 1)!)1^{n+1} = 0.841/(n + 1)!$ for even n and by $1/(n + 1)!)1^{n+1} = 1/(n + 1)!$ for odd n . A similar analysis works for $\sin x$ if the words “even” and “odd” are interchanged.