

10.3, #11. We multiple the two series:

$$\begin{aligned}
 e^t \cos t &= \left(1 + t + \frac{1}{2}t^2 + \frac{1}{6}t^3 + \dots\right)\left(1 - \frac{1}{2}t^2 + \frac{1}{24}t^4 + \dots\right) \\
 &= 1 + t + \left(\frac{1}{2}t^2 - \frac{1}{2}t^2\right) + \left(\frac{1}{6}t^3 - \frac{1}{2}t^3\right) + \left(\frac{1}{24}t^4 - \frac{1}{4}t^4 + \frac{1}{24}t^4\right) + \dots \\
 &= 1 + t - \frac{1}{3}t^3 - \frac{1}{6}t^4 + \dots.
 \end{aligned}$$

10.3, #15. We expand the three functions in series about $y = 0$:

$$\begin{aligned}
 \ln(1 + y^2) &= y^2 - \frac{1}{2}y^4 + \dots \\
 \sin(y^2) &= y^2 - \frac{1}{6}y^6 + \dots \\
 1 - \cos y &= \frac{1}{2}y^2 - \frac{1}{24}y^4 + \dots.
 \end{aligned}$$

So for y near 0, but $y \neq 0$, we have

$$\sin(y^2) > \ln(1 + y^2) > 1 - \cos y > 0.$$

10.3, #16. We expand the three functions in series about $\theta = 0$:

$$\begin{aligned}
 1 + \sin \theta &= 1 + \theta - \frac{1}{6}\theta^3 + \dots \\
 \cos \theta &= 1 - \frac{1}{2}\theta^2 + \dots \\
 \frac{1}{1 - \theta^2} &= 1 + \theta^2 + \dots.
 \end{aligned}$$

So for $\theta > 0$, we have

$$\cos \theta < \frac{1}{1 - \theta^2} < 1 + \sin \theta.$$

10.3, #21.

$$\begin{aligned} E &= \frac{kQ}{(R-1)^2} - \frac{kQ}{(R+1)^2} = \frac{kQ}{R^2} \left(\frac{1}{(1-1/R)^2} - \frac{1}{(1+1/R)^2} \right) \\ &= \frac{kQ}{R^2} \left(1 + 2\frac{1}{R} + 3\frac{1}{R^3} + \cdots - 1 + 2\frac{1}{R} - 3\frac{1}{R^3} + \cdots \right) \\ &= \frac{4kQ}{R^3} \left(1 + \frac{2}{R^2} + \frac{3}{R^4} + \cdots \right) \end{aligned}$$

10.3, #23.

$$\begin{aligned} V &= 2\pi\sigma(\sqrt{R^2 + a^2} - R) \\ &= 2\pi\sigma R(\sqrt{1 + (a/R)^2} - 1) \\ &= 2\pi\sigma R \left(1 + \frac{1}{2}(a/R)^2 + \cdots - 1 \right) \\ &= \frac{\pi\sigma a^2}{R} + \text{higher order terms.} \end{aligned}$$

10.3, #26.

$$\begin{aligned} F &= \frac{mgR^2}{(r+h)^2} = mg \frac{1}{(1+h/R)^2} \\ &= mg(1 - 2(h/r) + 3(h/R)^2 - + \cdots) \\ &= mg \end{aligned}$$