10.2, #12. If $f(x) = x^{-1}$, then $f'(x) = -x^{-2}$, $f''(x) = +2!x^{-3}$, and, in general, $f^{(n)}(x) = (-1)^n n! x^{-(n+1)}$. Evaluating these derivatives at 1 we have $f(1) = 1$, $f'(1) = -1$, $f''(1) = 2!$, and $f^{(n)}(1) = (-1)^n n!$. The taylor series is

$$1 - (x-1) + (x-1)^2 - (x-1)^3 + - \cdots + (-1)^n (x-1)^n + \cdots.$$

10.2, #19. $\sin\theta = \theta - \theta^3/3! + \theta^5/5! - \theta^7/7! + \cdots$. Thus

$$\frac{\theta - \sin\theta}{\theta^3} = \frac{\theta^3/3! - \theta^5/5! + \theta^7/7! - \cdots}{\theta^3} = 1/3! - \theta^2/5! + \theta^4/7! - \cdots.$$

This has limit 1/6 as $\theta \to 0$.

10.2, #20. $\tan\alpha = \alpha + \alpha^3/3 + \cdots$. Thus

$$\frac{\tan\alpha}{\alpha} = 1 + \alpha^2/3 + \cdots.$$

This has limit 1 as $\alpha \to 0$.

10.2, #21. $\sqrt{1+x}$ has Taylor series $1 + x/2 - x^2/8 + x^3/16 - 5x^4/128 + \cdots$ using 10.2, #7 or the binomial series with $p = 1/2$. Thus

$$\begin{aligned}\lim_{x\to 0} \frac{\sqrt{1+x} - 1}{x} &= \lim_{x\to 0} \frac{x/2 - x^2/8 + x^3/16 - 5x^4/128 + \cdots}{x} \\ &= \lim_{x\to 0} (1/2 - x/8 + x^2/16 - 5x^3/128 + \cdots) = 1/2.\end{aligned}$$

10.2, #28. The polynomials $P_n(x)$ for $n \le 4$ appproximate $\sqrt{1+x}$ well for $|x| \le 1/4$. This estimate was made using the following parameters for the window: $-1.4 \le x \le 3$, $-.5 \le y \le 2$. So the interval of convergence based on this observation would be $(-1/4, 1/4)$.

10.2, #30. $f'(0)$, $f''(0)$, $f'''(0)$, and $f^{(10)}(0)$ are related to the coefficients C_1, C_2, C_3, C_{10} of the Taylor series by the formula $f^{(j)}(0) = j!\,C_j$. Thus

$$f'(0) = 1, \quad f''(0) = 1, \quad f'''(0) = 2, \quad f^{(10)}(0) = 9! = 362880$$

10.2, #31. The derivatives asked for are $f'(0) =$ and $f^{(6)}(0)$. They are related to the coefficients C_1 and C_6, in the Taylor series by the formula in the previous problem, $f'(0) = 0$, since the coefficient of x is 0, and $f^{(6)}(0) = 6!/2! = 6 \cdot 5 \cdot 4 \cdot 3 = 360$.