10.1, #18. $P_3(x) = \sqrt{2} + \dfrac{(x-1)}{2\sqrt{2}} - \dfrac{(x-1)^2}{16\sqrt{2}} + \dfrac{(x-1)^3}{64\sqrt{2}}$ since

$$\begin{aligned}
f^{(0)}(x) &= (1+x)^{1/2}, & f^{(0)}(1) &= \sqrt{2},\\
f^{(1)}(x) &= (1/2)(1+x)^{-1/2}, & f^{(1)}(1) &= 1/2\sqrt{2},\\
f^{(2)}(x) &= -(1/2)(1/2)(1+x)^{-3/2}, & f^{(2)}(1) &= -1/8\sqrt{2},\\
f^{(3)}(x) &= (3/2)(1/2)(1/2)(1+x)^{-5/2}, & f^{(3)}(1) &= 3/32\sqrt{2}.
\end{aligned}$$

10.1, #19. $P_4(x) = \dfrac{1}{3} - \dfrac{(x-2)}{9} + \dfrac{(x-2)^2}{27} - \dfrac{(x-2)^3}{81} + \dfrac{(x-2)^4}{243}$ since

$$\begin{aligned}
f^{(0)}(x) &= (1+x)^{-1}, & f^{(0)}(2) &= 1/3,\\
f^{(1)}(x) &= (-1)(1+x)^{-2}, & f^{(1)}(2) &= -1/9,\\
f^{(2)}(x) &= (-1)^2 2!(1+x)^{-3}, & f^{(2)}(2) &= 2!/27,\\
f^{(3)}(x) &= (-1)^3 3!(1+x)^{-4}, & f^{(3)}(2) &= -3!/81,\\
f^{(4)}(x) &= (-1)^4 4!(1+x)^{-5}, & f^{(4)}(2) &= 4!/243.
\end{aligned}$$

10.1, #23–26. $a = f(0)$, $b = f'(0)$, and $c = f''(0)/2$. For each picture a is the y-intercept, b is positive or negative according as the graph is increasing or decreasing, and c is positive or negative according as the graph is concave-up or concave-down. There are 2^3 cases, the four here and four more you should try to draw.

10.2, #1–3. #1 is a Taylor series. #2 is not a Taylor series since it contains negative powers of x. #3 is not a Taylor series since the center of each additional term changes.

10.2, #7. The first four derivatives of $\sqrt{1+x}$ were calculated in 10.1, #2:

$$
\begin{aligned}
f^{(0)}(x) &= (1+x)^{1/2}, & f^{(0)}(0) &= 1,\\
f^{(1)}(x) &= (1/2)(1+x)^{-1/2}, & f^{(1)}(0) &= 1/2,\\
f^{(2)}(x) &= -(1/2)(1/2)(1+x)^{-3/2}, & f^{(2)}(0) &= -1/4,\\
f^{(3)}(x) &= (3/2)(1/2)(1/2)(1+x)^{-5/2}, & f^{(3)}(0) &= 3/8,\\
f^{(4)}(x) &= -(5/2)(3/2)(1/2)(1/2)(1+x)^{-7/2}. & f^{(4)}(0) &= -15/16
\end{aligned}
$$

For an arbitrary n

$$
\begin{aligned}
f^{(n)}(x) &= (-1)^{n-1}\frac{(2n-3)(2n-5)\cdots 3\cdot 1}{2^n}(1+x)^{-2n+1/2}\\
f^{(n)}(0) &= (-1)^{n-1}\frac{(2n-3)(2n-5)\cdots 3\cdot 1}{2^n}
\end{aligned}
$$

The Taylor series for $\sqrt{1+x}$ is

$$
\begin{aligned}
&1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}-\frac{5x^4}{128}+\cdots+(-1)^{n-1}\frac{(2n-3)(2n-5)\cdots 3\cdot 1}{2^n n!}x^n+\cdots\\
&1+\frac{1}{2}x-\frac{1\cdot 1}{2\cdot 4}x^2+\frac{1\cdot 1\cdot 3}{2\cdot 4\cdot 6}x^3-\frac{1\cdot 1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}x^4+-\cdots.
\end{aligned}
$$

10.2, #9. The first several derivatives of $(1-y)^{1/3}$ are

$$
\begin{aligned}
f^{(0)}(y) &= (1-y)^{1/3}, & f^{(0)}(0) &= 1\\
f^{(1)}(y) &= -(1/3)(1-y)^{-2/3}, & f^{(1)}(0) &= -1/3\\
f^{(2)}(y) &= -(2/3)(1/3)(1-y)^{-5/3}, & f^{(2)}(0) &= -2/9\\
f^{(3)}(y) &= -(5/3)(2/3)(1/3)(1-y)^{-8/3} & f^{(3)}(0) &= -10/27
\end{aligned}
$$

For arbitrary $n \geq 1$

$$
\begin{aligned}
f^{(n)}(y) &= -\frac{(3n-4)(3n-7)\cdots 5\cdot 2}{3^n}(1-y)^{-3n+1/3}\\
f^{(n)}(0) &= -\frac{(3n-4)(3n-7)\cdots 5\cdot 2}{3^n}
\end{aligned}
$$

The Taylor series for $(1-y)^{1/3}$ is

$$
1-\frac{y}{3}-\frac{y^2}{9}-\frac{5y^3}{81}-\cdots-\frac{(3n-4)(3n-7)\cdots 2}{3^n n!}y^n-\cdots
$$