

8.Rev, #6. A cut by a plane perpendicular to the x -axis at x meets the doughnut in an annulus with outer radius $3 + \sqrt{1 - x^2}$ and inner radius $3 - \sqrt{1 - x^2}$ for x in the range $-1 \leq x \leq 1$. The area of this annulus is $\pi\{(3 + \sqrt{1 - x^2})^2 - (3 - \sqrt{1 - x^2})^2\} = 12\pi\sqrt{1 - x^2}$.

The volume is $\int_{-1}^1 12\pi\sqrt{1 - x^2} dx = 12\pi \cdot \pi/2 = 6\pi^2$.

8.Rev, #9. The arclength of $y = e^x$ from $x = 1$ to $x = 2$ is given by the integral $\int_1^2 \sqrt{1 + e^{2x}} dx = 4.785$ by Simpson's rule. The substitution $w^2 = 1 + e^{2x}$ will lead eventually to the same result.

8.Rev, #15. An income stream of \$100 per year for 20 years, assuming 10% annual interest rate compounded continuously has a present value of

$$P = \int_0^{20} 100e^{-0.1t} dt = 100\left(\frac{-1}{0.1}\right)(e^{-2} - e^0) = \$864.66.$$

The future value of this amount is $B = \$864.66e^{0.1 \times 20} = \6389.06 . The balance will reach \$5000 in y years if

$$5000 = \int_0^y 100e^{0.1(y-t)} dt = 1000(e^{0.1y} - 1)$$

so $5 + 1 = e^{0.1y}$ and $0.1y = \ln 6$, so $y = 17.92$ years.

8.Rev, #18. If $\text{Prob}\{0 \leq t \leq 6\} = 0.10$, then

$$0.1 = \int_0^6 ce^{-ct} dt = 1 - e^{-6c}$$

so $e^{-6c} = 0.9$, $-6c = \ln 0.9$, and $c = 0.0176$. Then $\text{Prob}\{6 \leq t \leq 12\} = \int_6^{12} ce^{-ct} dt = e^{-6c}(1 - e^{-6c}) = 0.9(1 - 0.9) = 0.09$.

8.Rev, #25. The pressure at depth y below the surface is $62.4y \text{ lb/ft}^2$. The bottom has area $\pi \text{ ft}^2$ and the force on the bottom, at depth 2ft, is $62.4 \times 2 \times \pi = 392 \text{ lb}$. The area of the cylindrical region of the side between depth y and $y + \Delta y$ is $2\pi \Delta y \text{ ft}^2$ and the force on this region is approximately $62.4y 2\pi \Delta y \text{ lb}$. The total force on the side is

$$\int_0^2 62.4y 2\pi dy = 784 \text{ lb}.$$

the total force on side and bottom is 1176 lb.

8.Rev, #26. The apple is about 3.5 inches high. Slicing horizontally we get shapes which are approximately disks, so the volume is about $\pi \int_0^{3.5} r(y)^2 dy$ where $r(y)$ is the approximate radius. From the graph of the apple we have

y	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
$r(y)$	0.5	1.5	1.75	2.0	2.25	2.25	2.0	1.5

Using the trapezoid rule we find

$$V = 0.5\pi\left\{\frac{1}{2}0.5^2 + 1.5^2 + 1.75^2 + 2.0^2 + \cdots + 2.0^2 + \frac{1}{2}1.5^2\right\} = 38.8 \text{ in}^3.$$

The weight is $38.8 \times 0.03 = 1.16 \text{ lb}$ and the cost is $1.16 \times 80 = 93 \text{ cents}$.