8.6, #6. The fraction of the population making between \$20000 and \$50000 is $P(50) - P(20) = 99 - 75 = 24\%$. The median income is the x for which $P(x) = 50\%$, that is, $x = \$12600$. The table for $P(x)$ shows the fastest increase in the range $7.8 \le x \le 12.6$, so the density function $p(x)$ has its maximum in this interval, say at $x = 9$. This point would also be an inflection point of $P(x)$.

8.6, #7. The probability density function for a gap in time of x seconds between cars on a certain freeway is

$$p(x) = \begin{cases} 0 & x < 0 \\ ae^{-0.122x} & x \ge 0. \end{cases}$$

In order to have $\int_0^\infty p(x)\,dx = 1$, we must have $a = 0.122$. The cumulative distribution

$$P(t) = \int_0^t 0.122e^{-0.122x}\,dx = 1 - e^{-.0122t}.$$

The median gap T satisfies the condition $P(T) = 1/2$, so $T = \ln 2/0.122 = 5.68$sec. The mean is $\bar{t} = \int_0^\infty tp(t)\,dt = 8.2$sec.

8.6, #9. A distribution of velocities is given by $p(v) = av^2e^{-mv^2/(2kT)}$ for $v \ge 0$ and $p(v) = 0$ for $v < 0$. The value of a is determined by the condition $\int_0^\infty p(v)\,dv = 1$. We use the change of variables $x = \sqrt{\frac{m}{kT}}v$ to simplify

$$\int_0^\infty p(v)\,dv = a\left(\frac{kT}{m}\right)^{3/2}\int_0^\infty x^2e^{-x^2/2}\,dx.$$

Using integration by parts,

$$\int x^2e^{-x^2/2}\,dx = -xe^{-x^2/2} + \int e^{-x^2/2}\,dx.$$

From the normal distribution we have

$$\int_0^\infty x^2 e^{-x^2/2}\,dx = \sqrt{\frac{2}{\pi}},$$

so $\int_0^\infty p(v)\,dv = a\left(\frac{kT}{m}\right)^{3/2}\left\{0 + \sqrt{\frac{2}{\pi}}\right\}$. Therefore

$$a = \sqrt{\frac{2}{\pi}}\left(\frac{m}{kT}\right)^{3/2}.$$

The mean speed is

$$\bar{v} = \int_0^\infty vp(v)\,dv = a\int_0^\infty v^3 e^{-mv^2/(2kT)}\,dv = 2a\left(\frac{kT}{m}\right)^2 = \sqrt{\frac{8}{\pi}}\sqrt{\frac{kT}{m}} = 1.596\sqrt{\frac{kT}{m}},$$

(using integration by parts).

The median speed V satisfies

$$1/2 = \int_0^V p(v)\,dv = a\int_0^V v^2 e^{-v^2/(2kT)}\,dv.$$

Letting $x = \sqrt{\frac{m}{kT}}v$ as above we get

$$1/2 = \sqrt{\frac{2}{\pi}}\int_0^{V\sqrt{m/(kT)}} x^2 e^{-x^2/2}\,dx.$$

This gives

$$\int_0^{V\sqrt{m/(kT)}} x^2 e^{-x^2/2}\,dx = \sqrt{\frac{\pi}{8}} = 0.626657.$$

Using Simpson's rule to approximate the integral we find

u	1	2	1.5	1.6	1.53	1.538
$\int_0^u x^2 e^{-x^2/2}\,dx$	0.2491	0.9256	0.5989	0.6711	0.6207	0.62653

Hence $V\sqrt{m/(kT)} = 1.538$, so the median value $V = 1.538\sqrt{\frac{kT}{m}}$

The maximum of $p(v)$ occurs at a values of v for which $p'(v) = 0$. (This value of v is called the mode.) Taking the derivative and solving for v we find $v = \sqrt{\frac{2kT}{m}} = 1.414\sqrt{\frac{kT}{m}}$. The mode, median, and mean are in increasing order and are each proportional to $\sqrt{T}$.

8.Rev, #1. Let S be the solid obtained by rotating the given region around the x-axis, $y = 0$. Cuts perpendicular to the x-axis meet S in disks, the cut through x has radius $\sqrt{x}$ and so has area πx. A Riemann sum approximating the volume os S is $\Sigma_{i=1}^n \pi x_i \Delta x$ where $x_i = i/n$. The volume is given by $\displaystyle\int_0^1 \pi x\,dx = \frac{\pi}{2}$.

8.Rev, #2. Rotating the same region around the horizontal line $y = 1$ the cuts are disks of radius $1 - \sqrt{x}$, their areas are $\pi(1 - \sqrt{x})^2$ and the volume is

$$\pi \int_0^1 (1 - \sqrt{x})^2\,dx = \pi \int_0^1 1 - 2\sqrt{x} + x\,dx = \frac{\pi}{6}.$$

Rotating around the y-axis, $x = 0$, cuts perpendicular to the y-axis are disks of radius y^2 and area πy^4. The volume is

$$\int_0^1 \pi y^4\,dy = \frac{\pi}{5}.$$

8.Rev, #3. For the line $y = ax$ A cut perpendicular to the x-axis at x meets the cone in a disk of radius bx/l which has area $\pi b^2 x^2/l^2$. The volume is

$$\pi \int_0^l b^2 x^2/l^2\,dx = \frac{1}{3} b^2 l.$$