8.5, #5. Let's say "most" means something like 65% of the plants. The cumulative distribution functions of the plants with fertilizer A, with no ferilizer, and with fertilizer B have value .65 when the heights of the plants are respectively .9, 1.1, and 1.3 meters. So fertilizer A stunts the growth of most plants, while fertilizer B promotes the growth of most plants.

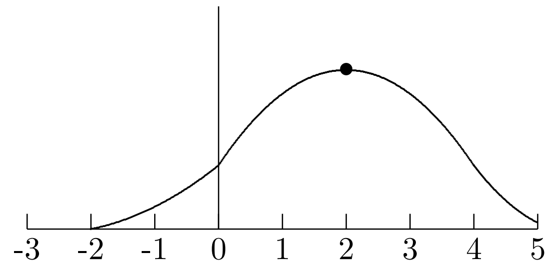
8.5, #6. The density function for the duration of phones calls is $0.4e^{-0.4x}$. The fraction of calls lasting between 1 and 2 minutes is

$$\int_1^2 0.4e^{-0.4x}\,dx = [-e^{-0.4x}]_1^2 = 0.221 \text{ or } 22.1\%.$$

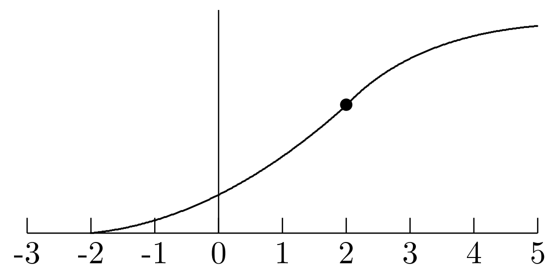
For calls lasting less than one minute, we integrate form 0 to 1 and get 33.0%. For calls lasting longer than 3 minutes, we integrate from 3 to infinity and get 30.1%. The cumulative distribution is

$$P(x) = \int_1^x 0.4e^{-0.4u}\,du = 1 - e^{-0.4x} \quad \text{for} \quad x \geq 0.$$

8.5, #8. The density function $p(t)$ would be the % of times per minute the bus is t minutes late, where negative t would mean early arrival $-t$ minutes ahead of schedule. So $p(t)$ would have a graph of the following shape:



The area under the graph of $p(t)$ between $t = 2$ and $t = 4$ represents the percentage of times the bus is late by 2 to 4 minutes. The cumulative distribution function $P(t)$ has a graph of the following shape:



The area under the graph of $p(t)$ between $t = 2$ and $t = 4$ is $P(4) - P(2)$ since P is an antiderivative of $p(t)$. $P(t)$ is concave-up when $p(t)$ is increasing, concave-down when $p(t)$ is decreasing. An inflection point of $P(t)$ is a critical point of $p(t)$. In this problem $p(t)$ has only one critical point and $P(t)$ has only one inflection point. $P(t)$ is increasing at a faster and faster rate up to the inflection point, increasing at a slower and slower rate after the inflection point.

8.6, #1. The graph in figure 8.44 is linear from $x = 2$ to $x = 6$ and from $x = 6$ to $x = 8$. The area an be computed using the formula for the area of a trapezoid,

$$\text{Area} = (1/2)(0.08 + 0.24)(6 - 2) + (1/2)(0.24 + 0.12)(8 - 6) = 1.$$

The graph is a density function (for proportion of days with a given catch of fish), so the total area represents the total fraction, 1, of days considered.

8.6, #2. Since p is zero outside the interval $[2, 8]$, the average daily catch is

$$\int_2^8 xp(x)\,dx = \int_2^6 x\left\{\frac{0.24 - 0.08}{6 - 2}(x - 2) + 0.08\right\}dx + \int_6^8 x\left\{\frac{0.12 - 0.24}{8 - 6}(x - 8) + 0.12\right\}dx = 5.253\text{tons}.$$

The integrands are the linear functions whose graphs are pictured in figure 8.44.

8.6, #4. We are asked to approximate

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(x-15)^2/2}\, dx.$$

The difficulties are that the integral is improper and thta there is no elementary antiderivative. The integrand is largest near $x = 15$. We compute

$$\frac{1}{\sqrt{2\pi}} \int_{10}^{20} e^{-(x-15)^2/2}\, dx.$$

by Simpson's rule taking 4, 8, 16, and 32 subdivisions:

$$S_4 = 0.9716, \quad S_8 =).9999, \quad S_{16} = 1 - 6 \cdot 10^{-7}, S_{32} = 1 - 6 \cdot 10^{-7}.$$

To estimate the integral for 20 to infinity substitute $u = x - 15$ and note $e^{-u^2/2} < e^{-2.5u}$ for $u \geq 5$:

$$0 < \frac{1}{\sqrt{2\pi}} \int_{20}^{\infty} e^{-(x-15)^2/2}\, dx = \frac{1}{\sqrt{2\pi}} \int_{5}^{\infty} e^{-u^2/2}\, du < \frac{1}{\sqrt{2\pi}} \int_{5}^{\infty} e^{-2.5u}\, du = 6 \cdot 10^{-7}.$$

Similarly

$$0 < \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{10} e^{-(x-15)^2/2}\, dx < 6 \cdot 10^{-7}.$$

So we estimate

$$1 - 6 \cdot 10^{-7} < \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(x-15)^2/2}\, dx, < 1 + 6 \cdot 10^{-7}.$$