8.4, #9. Slice the interval $[0,T]$ into subintervals of width Δt. The profit in 1000's of dollars over $[t, t+\Delta t]$ is $(2-0.1t)\Delta t$. This profit has present value $(2-0.1t)\Delta t e^{-0.1t}$. Thus $\sum (2-0.1t)e^{-0.1t}\Delta t$ summed over subintervals approximates the present value of the total profit earned over $[0,T]$. The actual present value is $P(T) = \int_0^T (2-0.1t)e^{-0.1t}\,dt$. Its derivative $P'(T) = (2-0.1T)e^{-0.1T}$ is 0 when $T = 20$. This gives the global maximal of $P(T)$ since $P'(T)$ is positive on $[0,20)$ and negative on $(20,\infty)$. Formula #14 on page 366 gives

$$P(20) = \int_0^{20} (2-0.1t)e^{-0.1t}\,dt = (t-10)e^{-0.1t}\Big|_0^{20} = 11353.35 \text{ dollars.}$$

8.4, #11. The demand function $D(q)$ (in thousands of dollars) is the linear function $D(q) = 980 - (1.4)(q-50) = -1.4q + 1050$ (since $D(50) = 980$ and $D(350) = 560$). The equilibrium price and equilibrium quantity are $p^* = 560$ and $q^* = 350$. So the consumer surplus is the area of the triangle formed by the p-axis, the line $D(q) = -1.4q + 1050$, and the line $p = 560$. This is \$85,750,000 dollars.

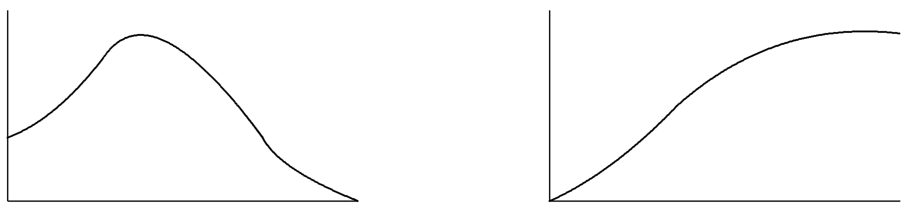
8.4, #12. The integral $\int_0^{q^*} p^*\,dq = p^*q^*$ is the total gross revenue to producers if thy sell the equilibrium quantity q^* for the equilibrium price p^*, which is the least price for which they will be willing to produce the quantity q^*.

In problem #13 below we see that $\int_0^{q^*} S(q)\,dq$ is the total gross revenue to producers if they sell each new unit of goods at the least price$S(q)$ for which they are willing to produce that total number of units.

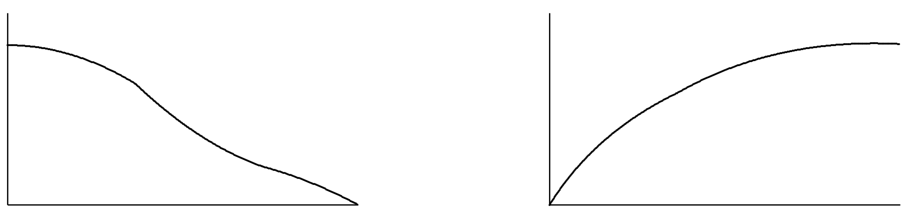
So $\int_0^{q^*} (p^* - S(q))\,dq$ is teh extra revenue produced by setting a fixed price at the level p^* needed to ensure the production of the quantity q^*.

8.4, #13. The supply function $p = S(q)$ is the price per unit producers will take when the quantity q is produced. Slice the interval $[0, q^*]$ into subintervals of width Δq. Assume that producers sell at the minimal price they are willing to take. Then the gross for producers on the subinterval $[q, q + \Delta q]$ is about $S(q)\Delta q$. Summing over all subintervals gives the approximation $\sum S(q)\Delta q$ for the total gross for producers up to the equilibrium quantity. This is a Riemann sum for $\int_0^{q^*} S(q)\,dq$. So the the integral is the total gross for producers if they sell at the minimal price they are willing to take and that minimal price is greater or equal to the equilibrium price.

8.5, #1. The density and cumulative distribution functions of a population with a large middle class might have the following graphs:



8.5, #2. The density and cumulative distribution functions of a population with a small middle class, a small upper classe, and many poor people might have the following graphs:



8.5, #3. The density and cumulative distribution functions of a population with a small middle class, and many rich and poor people might have the following graphs: