8.4, #2. The future value of the installment plan 3 years from now when the final payment is made is

$$5000 + 5000e^{0.08} + 5000e^{2(0.08)} + 5000e^{3(0.08)} = 22640.24$$

The future value of \$25000 paid 3 years from now is \$25000. Thus the lump payment is preferable. Alternatively, one can calculate the present values of the two options to see the lump payment is preferable.

8.4, #3. The present value of the income stream is

$$\begin{aligned}\int_0^{15} 3000e^{-0.06t}\,dt &= 3000\Big(\frac{e^{-0.06t}}{-0.06}\Big)\Big|_0^{15}\\ &= 29671.52.\end{aligned}$$

The future value of the income stream is

$$\begin{aligned}\int_0^{15} 3000e^{0.06(15-t)}\,dt &= 3000e^{0.9}\int_0^{15} e^{-0.06t}\,dt\\ &= 3000e^{0.9}\Big(\frac{e^{-0.06t}}{-0.06}\Big)\Big|_0^{15}\\ &= 72980.16\end{aligned}$$

8.4, #5. If P dollars is deposited at a continuous each year into the account, the future value of the stream in 10 years time is

$$\int_0^{10} Pe^{0.1(10-t)}\,dt = Pe\int_0^{10} e^{-0.1t}\,dt = -Pe\frac{e^{-0.1t}}{0.1}\Big|_0^{10} = 10Pe(1-e^{-1}).$$

This is \$100000 if $P = 10000/(e-1) = 5819.77$. The future value at the end of 10 years of a lump sum of L dollars deposited now is $Le^{(0.1)\times 10} = Le$. This is \$100000 if $L = 100000/e = 36787.94$.

8.4, #7. In (a) the future value of the deposit stream at time T is

$$\int_0^{T} 1000e^{0.05(T-t)}\,dt = -20000e^{0.05(T-t)}\Big|_0^{T} = 20000(e^{0.05T}-1).$$

$20000(e^{0.05T}-1) = 10000$ when $e^{0.05T} = 1.5$, that is , when $0.05T = \ln 1.5$ and $T = 8.11$ years. In (b) the future value of the deposit stream at time T is

$$2000e^{0.05T} + 20000(e^{0.05T}-1) = 22000e^{0.05T} - 20000.$$

$22000e^{0.05T} - 20000 = 10000$ when $e^{0.05} = 32/22 = 1.3636$, that is, when $0.05T = 0.3102$ and $T = 6.20$ years.