8.3, #9. A spherical planet of radius R and density ρ has volume $V = (4/3)\pi R^3$ and mass $M = (4/3)\pi R^3 \rho$. The escape velocity v for an object from this planet is given in Example 5 on page 441: $v = \sqrt{2GM/R}$. In particular, $v = \sqrt{8G\pi R^2 \rho/3}$ and v is proportional to R and to $\sqrt{\rho}$.

8.3, #11. Slice the rectangular wall of the dam into horizontal strips of width Δy. A strip y feet from the top of the dam has area $1000\Delta y$ ft^2. The pressure at that level is $62.4y$ lb/ft^2. So the strip exerts a force of $62.4y \times 1000\Delta y$ pounds. Summing over all strips gives $\sum 62400 y \Delta y$, a Riemann sum for $\int_0^{50} 62400 y\, dy$. The integral has value $31200(50)^2 = 78{,}000{,}000$ pounds.

8.3, #14. The car moves with velocity $v(t) = 50t/(t+1)$ mph at time t. On the time interval $2 \le t \le 3$, its velocity increases from $100/3 = 33.33$ mph to $150/4 = 37.5$ mph. So the formula $25 + 0.1v$ giving miles per gallon holds for this interval of time. Slice the interval $[2, 3]$ of time into pieces of width Δt. During the piece of time from t to $t + \Delta t$, the car goes approximately $v(t)\Delta t$ miles using

$$\frac{v(t)\Delta t}{25 + 0.1v(t)} = \frac{50t\,\Delta t}{(t+1)\,(25 + 5t/(t+1))} = \frac{10t\,\Delta t}{6t+5}$$

gallons of gasoline. The total amount of gasoline used is

$$\begin{aligned} 10\int_2^3 \frac{t\,dt}{6t+5} &= 10\int_2^3 \Big(\frac{1}{6} - \frac{5}{6(6t+5)}\Big)\,dt \\ &= 10\Big(\frac{t}{6} - \frac{5}{36}\ln(6t+5)\Big)\Big|_2^3 \\ &= (5/3) - (25/18)(\ln 23 - \ln 17) = 1.2468 \text{ gallons.} \end{aligned}$$

8.3, #15. Slice the rod into pieces of width Δx. Since the density of the rod is 5/3 kg/m, the mass of each piece is $5/3\Delta x$. A piece x meters from the axis of rotation moves with velocity $2x$ meters/sec. So its kinetic energy is $(1/2)(5/3)(2x)^2\,\Delta x = (10/3)x^2\,\Delta x$. The total energy of the rod is

$$\int_{-3}^{3} \frac{10x^2}{3}\,dx = 10/9x^3\Big|_{-3}^{3} = 60\text{kg m}^2/\text{sec}^2.$$

8.3, #16. Slice the record into concentric rings of width Δr. A ring of radius r has approximate area $2\pi r\Delta r$. Since the record has area $(10)^2\pi = 100\pi$ cm^2, it follows the ring of radius r has approximate mass $(2\pi r\Delta r/100\pi)50 = r\Delta r$. Now consider the velocity of a point on the ring as the record spins. In 60 seconds, the record has completed $33\frac{1}{3}$ revolutions. With each revolution the point has gone $2\pi r$ cm. Thus the point has a velocity of $(33\frac{1}{3})\cdot(2\pi r)/60 = (10/9)\pi r$ cm/sec. So the ring has velocity $(10/9)\pi r$ cm/sec and hence kinetic energy $(1/2)r\Delta r[(10/9)\pi r^2]^2$. Summing over all rings gives a Riemann sum for

$$\int_{0}^{10} \frac{50\pi^2 r^3}{81}\,dr = \frac{25\pi^2 r^4}{162}\Big|_{0}^{10} = 15231 \text{ ergs},$$

where an erg=gram-cm/sec^2.

8.3, #17. Slice the rod into pieces of width Δx. Since the rod has length ℓ and mass M, the mass of each piece is $M\,\Delta x/\ell$. Consider a piece of the rod at a distance $x+a$ from the particle. So if $x=0$, the piece is at the end of the rod nearest the particle, and if $x=\ell$, the piece is at the end of the rod farthest from the particle. The gravitational force between the piece and the particle is $(GMm\,\Delta x)/(\ell(a+x)^2)$. Thus the total force is

$$\begin{aligned}\frac{GMm}{\ell}\int_0^\ell \frac{dx}{(a+x)^2} &= -\frac{GMm}{\ell}\frac{1}{a+x}\Big|_0^\ell \\ &= -\frac{GMm}{\ell}\Big(\frac{1}{a+\ell}-\frac{1}{a}\Big) \\ &= \frac{GMm}{\ell}\frac{\ell}{a(a+\ell)} = \frac{GMm}{a(a+\ell)}.\end{aligned}$$

8.3, #18. Slice the first rod into pieces of width Δx. The mass of each piece is $M_1\,\Delta x/\ell_1$ (as computed in #17. Consider a piece of the first rod at a distance of $x+a$ from the near end of the second rod. By the solution of #17 the gravitational force between this piece and the second rod is $(GM_2M_1\,\Delta x)/(x+a)(x+a+\ell_2)$. Thus the total force between the two rods is

$$GM_1M_2\int_0^{\ell_1}\frac{dx}{(x+a)(x+a+\ell_2)} = G\frac{M_1}{l_1}\frac{M_2}{l_2}\ln\frac{(a+l_1)(a+l_2)}{a(a+l_1+l_2)}$$