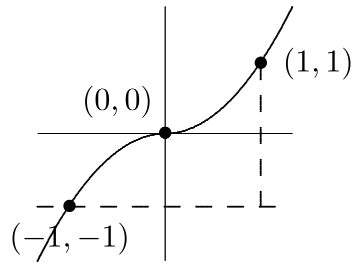
8.2, #6. Slice the solid by planes perpendicular to the axis of rotation. The solid is gotten by rotating the finite region enclosed by the graphs of $y = x^3$, $y = -1$, and $x = 1$. This region is illustrated below.



The graph of x^3 meets the line $y = -1$ at $(-1, -1)$ and the line $x = 1$ at $(1, 1)$.) A slice meeting the axis at $[x, x + \Delta x]$ has the approximate shape of a disk of radius $x^3 + 1$ and thickness Δx. Since the disk has volume $\pi(x^3 + 1)^2 \, \Delta x$, the total volume of all slices is $\sum \pi(x^3 + 1)^2 \, \Delta x$ and the solid has volume

$$\int_{-1}^{1} \pi(x^6 + 2x^3 + 1) \, dx = \pi\left(\frac{x^7}{7} + \frac{x^4}{2} + x\right)\Bigg|_{-1}^{1} = \frac{16\pi}{7} = 7.1808.$$

8.2, #7. Slice the solid by planes perpendicular to the axis of rotation. A slice meeting the axis at $[y, y + \Delta y]$ has the approximate shape of a disk of radius $1 - y^2$ (since $y = \sqrt{x}$) and thickness Δy. The volume of the disk is $\pi(1 - y^2)^2 \Delta y$, so the total volume of all slices is $\sum \pi(1 - y^2)^2 \Delta y$. Thus the solid has volume

$$\int_{0}^{1} \pi(1 - 2y^2 + y^4) \, dy = \pi\left(y - \frac{2}{3}y^3 + \frac{y^5}{5}\right)\Bigg|_{0}^{1} = \frac{8\pi}{15} = 1.6755.$$

8.2, #11. Slice the solid by planes perpendicular to the axis of rotation. A slice meeting the x-axis at x has the approximate shape of an annulus with outer radius 7, inner radius $7-e^x$, and thickness Δx. The annulus has volume $\pi(7^2-(7-e^x)^2)\Delta x = \pi(14e^x - e^{2x})\Delta x$. So the solid has volume

$$\int_0^1 \pi(14e^x - e^{2x})\,dx = \pi(14e^x - (e^{2x}/2))\Big|_0^1 = \pi(14e - 14 - (e^2-1)/2) = 65.5381$$

8.2, #12. Slice the solid by planes perpendicular to the x-axis. A slice meeting the axis at x has the approximate shape of a rectangular box of side lengths e^x, e^x, and Δx (since the line segment connecting the boundary points $(x, 0)$ and (x, e^x) of the region has length e^x). Since the box has volume $e^{2x}\Delta x$, the solid has volume

$$\int_0^1 e^{2x}\,dx = (e^{2x}/2))\Big|_0^1 = (e^2-1)/2) = 3.1945.$$

8.2, #15. Slice the tree into 6 circular cylinders of length 20 feet. The area of a circle of circumference C is $C^2/4\pi$. The corresponding approximations for the volume in cubic feet of the tree are

$$\begin{aligned}
\text{LEFT} &= \frac{20}{4\pi}(26^2+22^2+19^2+14^2+6^2+3^2) = 2804.31\\
\text{RIGHT} &= \frac{20}{4\pi}(22^2+19^2+14^2+6^2+3^2+1^2) = 1730.01\\
\text{TRAP} &= \frac{1}{2}(\text{LEFT}+\text{RIGHT}) = 2267.16
\end{aligned}$$

8.2, #18. This graph is a quarter of the circle of radius 2, so the arclength will be π. The function $f(x) = \sqrt{4-x^2}$ for $0 \le x \le 2$. Its derivative is $-x/\sqrt{4-x^2}$ so

$$1 + (f'(x))^2 = 1 + \frac{x}{4-x^2} = \frac{4}{4-x^2}$$

and the integral is

$$\int_0^2 \frac{2}{\sqrt{4-x^2}}\,dx$$

From tables or using the substitution $x = 2\sin\theta$ we find the arc lenght to be π.

8.2, #19. We have $f(x) = x^{3/2}$, so $f'(x) = (3/2)x^{1/2}$ and the integral is

$$\int_0^2 \sqrt{1 + (9/4)x}\,dx = (8/27)((11/2)^{3/2} - 1) = 3.5255.$$

This value can be checked by Simpson's rule.

8.2, #21. $y = \dfrac{e^x + e^{-x}}{2}$, $y' = \dfrac{e^x - e^{-x}}{2}$. So

$$\sqrt{1 + (y')^2} = \sqrt{1 + \frac{e^{2x} - 2 + e^{-2x}}{4}} = \sqrt{\frac{e^{2x} + 2 + e^{-2x}}{4}} = \frac{e^x + e^{-x}}{2}.$$

The length of the catenary on $[-1, 1]$ is

$$\frac{1}{2}\int_{-1}^{1} (e^x + e^{-x})\,dx = \frac{1}{2}(e^x - e^{-x})\Big|_{-1}^{1} = e - e^{-1} = 2.35.$$