8.1, #1. Break up the rod by slicing it with planes perpendicular to the direction of the rod. A slice of thickness Δx at a point x meters from the left end of the rod has approximately mass $\rho(x)\,\Delta dx = (2+6x)\,\Delta x$ g/m. So the rod has total mass

$$\int_0^2 (2+6x)\,dx = 2x + 3x^2\Big|_0^2 = 16 \text{ grams.}$$

Any Riemann sum for this integral answers the first part of the question.

8.1, #2. The moment of the rod with respect to its left end is

$$\int_0^2 x(2+6x)\,dx = x^2 + 2x^3\Big|_0^2 = 20.$$

Thus moment/mass$= 20/16 = 1.25$ and the center of mass of the rod is 1.25 meters from the left end.

8.1, #3. We are given a function $\rho(x)$m for the density of cars (in cars per mile). Between x and $x+\Delta x$ there are appoximately $\rho(x)\Delta x$ cars. The total number of cars in the twenty miles from $x=0$ to $x=20$ is approximately

$$\Sigma_{i=0}^{n-1}\rho(x_i)\Delta x$$

where $x_i = 20i/n$ and $\Delta x = 20/n$. This is a Riemann sum for the integral

$$300\int_0^{20} 2+\sin(4\sqrt{x+0.15})\,dx$$

Two Simpson's approximations are $S_{20} = 11505.3$ and $S_{40} = 11512.0$. The integral can be evaluated in elementary terms by substituting $u^2 = 16(x+0.15)$ leading to the answer 11512.85 cars.

8.1, #4. The city has radius 3 miles since the population density $10000(3-r)$ is zero when $r=3$. Partition the city into concentric rings around its center, where each ring has thickness Δr. Since the ring of inner radius r and thickness Δr has approximately area $2\pi r\Delta r$, the population of the ring is $2\pi r(10000(3-r))\Delta r$. The total population is then

$$\int_0^3 2\pi 10000(3-r)r\,dr = 20000\pi\Big(\frac{3r^2}{2}-\frac{r^3}{3}\Big)\Big|_0^3 = 20000\pi(9/2) = 282743.$$

8.1, #7. Let x be the distance from the side of length 3. Then $0 \le x \le 3$. The strip between x and $x+\Delta x$ has area $5\Delta x$ and mass approximately $5/(1+x^4)\,\Delta x$. The total mass is approximately

$$\Sigma_{i=0}^{n-1} 5/(1+x^4)\,\Delta x.$$

This is a Riemann sum for the integral

$$\int_0^3 5/(1+x^4)\,dx.$$

There are various ways to get the requested precision. The Simpson approximations $S_2 = 5.664$, $S_4 = 5.486$, $S_8 = 5.492$, and $S_16 = 5.492$ indicate the mass is 5.50. [The integral can be found in elementary terms by a use of partial fractions.]

8.1, #11. The cross-section of the pipe is a disk of radius 1 inch. At a point r inches from the center of the disk, water flows at the rate of $10(1-r^2)$ inches/sec. Partition the cross-section into concentric rings of thickness Δr. The flow of water across a ring of inner radius r and thickness Δr is approximately $(2\pi r\Delta r)\times 10(1-r^2)$ since the area of the ring is approximately $2\pi r\Delta r$. Thus the total flow of water across the disk is

$$20\pi\int_0^1 (r-r^3)\,dr = 20\pi\Big(\frac{r^2}{2}-\frac{r^4}{4}\Big)\Big|_0^1 = 5\pi \text{ inches}^3/\text{sec}.$$

8.2, #2. Let z be vertical displacement from the center of the sphere, so $-r \le z \le r$. The horizontal cut at height z is a disk of radius $\sqrt{r^2 - z^2}$ which has area $\pi(r^2 - z^2)$. The slice between the cuts at z and $z + \Delta z$ has volume approximately $\pi(r^2 - z^2)\Delta z$. The resulting sum is a Riemann sum for the integral

$$\int_{-r}^{r} \pi(r^2 - z^2)\, dz = \frac{4}{3}\pi r^3.$$

8.2, #3. Again a horizontal cut at height z above the base will be a disk of radius $(3-z)/3$ with area $\pi(3-z)^2/9$. The slice between z and $z+\Delta z$ has volume approximately $\pi(3-z)^2/9\, \Delta z$ so the total volume is given by the integral $\int_0^3 \pi(3-z)^2/9\, dz = \pi$.

8.2, #4. If the cone hs height h and base radius r then the radius at height z is $b(1-z/h)$. The integral in problem #3 becomes

$$\int_0^b \pi b^2(1 - z/h)^2\, dz = \frac{1}{3}\pi b^2 h.$$

8.2, #5. Slice the solid by planes perpendicular to the axis of rotation. A slice meeting the axis at $[x, x + \Delta x]$ has the approximate shape of a disk of radius $\sin x$ and thickness Δx. The volume of the disk is $\pi \sin^2 x\, \Delta x$, so the total volume of all the slices is $\sum \pi \sin^2 x\, \Delta x$. Thus the solid has volume

$$\int_0^{\pi} \pi \sin^2 x\, dx = \pi\Big(-\frac{1}{2}\sin x \cos x + \frac{1}{2}x\Big)\Big|_0^{\pi} = (\pi^2/2) = 4.9348,$$

using formula 17 on page 367.