7.Rev, #20. Use formula 25 on page 367 to get

$$\int \frac{5x+6}{x^2+4}\,dx = \frac{5}{2}\int \frac{2x}{x^2+4}\,dx + 6\int \frac{dx}{x^2+4} = \frac{5}{2}\ln(x^2+4) + 3\arctan\frac{x}{2} + C.$$

7.Rev, #25. $\displaystyle\int \frac{(u+1)^3}{u^2}\,du = \int \frac{u^3+3u^2+3u+1}{u^2}\,du = \frac{1}{2}u^2 + 3u + 3\ln|u| - \frac{1}{u} + C.$

7.Rev, #32. $u = x^2$, $du = 2x\,dx$ changes $\displaystyle\int \frac{x\,dx}{\sqrt{1-x^4}}$ to $\displaystyle\frac{1}{2}\int \frac{du}{\sqrt{1-u^2}}$.

7.Rev, #35.

$$\int \frac{e^x}{1+e^x}\,dx = \ln(1+e^x) + C$$

$$\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{dw}{1+w^2} = \arctan w + C = \arctan e^x + C$$

(substituting $w = e^x$)

$$\int \frac{dx}{1+e^x} = \int \frac{dw}{w(1+w)} = \ln|w| - \ln|1+w| + C = x - \ln(1+e^x) + C$$

(substituting $w = e^x$ and using formula 26 on page 367)

7.Rev, #36. The averages over one cycle in ascending order are

$$\begin{aligned}
\frac{1}{2\pi}\int_0^{2\pi} \cos t\,dt &= 0,\\
\frac{1}{2\pi}\int_0^{2\pi} \cos^2 t\,dt &= \frac{1}{2\pi}\frac{1}{2}(\cos x\sin x + x)\Big|_0^{2\pi} = \frac{1}{2}\\
\frac{1}{2\pi}\int_0^{2\pi} |\cos t|\,dt &= 4\times\frac{1}{2\pi}\int_0^{\pi/2}\cos t\,dt \quad \text{(by symmetry)}\\
&= \frac{2}{\pi}\sin t\Big|_0^{\pi/2} = \frac{2}{\pi}
\end{aligned}$$

The first average is clearly zero becuase of the oscillation of $\cos x$. The other averages are positive since $\cos^2 t$ and $|\cos t|$ are non-negative. Moreover, $\cos^2 t = |\cos t|^2 \le |\cos t|$ since $|\cos t| \le 1$. So the average for $\cos^2 t$ is less than that for $|\cos t|$.

7.Rev, #40. $y = x^2$ and $y = 6 - x$ intersect at $(2,4)$. Since a vertical strip of the shaded region lying over x has length $6 - x - x^2$, the average vertical height of the shaded region is

$$\frac{1}{2}\int_0^2 (6 - x - x^2)\,dx = \frac{1}{2}\left(6x - \frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^2 = \frac{1}{2}(12 - 2 - (8/3)) = 11/3$$

7.Rev, #51. The integral is improper since $1/\sqrt{t+5}$ is undefined at $t = -5$. The integral converges since

$$\lim_{b\to -5^+}\int_b^{10} \frac{dt}{\sqrt{t+5}} = \lim_{b\to -5^+} 2\sqrt{t+5}\Big|_b^{10} = \lim_{b\to -5^+} 2(\sqrt{15} - \sqrt{b+5}) = 2\sqrt{15}.$$