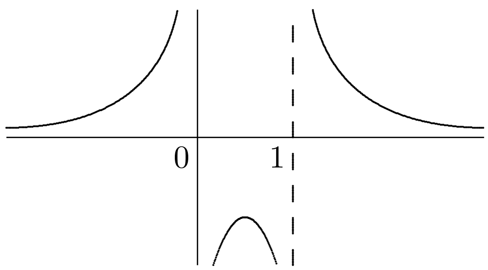
7.8, #4. $\displaystyle\int_b^0 \frac{e^x}{1+e^x}\,dx = \ln(1+e^x)\Big|_b^0 = \ln 2 - \ln(1+e^b)$. This has limit $\ln 2$ as $b \to -\infty$.

7.8, #17. $\displaystyle\int_2^b \frac{1}{x\ln x}\,dx = \ln(\ln x)\Big|_2^b = \ln(\ln b) - \ln(\ln 2)$ (use the substitution $w = \ln x$.) This diverges as $b \to \infty$.

7.8, #20. $\displaystyle\int_2^b \frac{1}{x(\ln x)^2}\,dx = -\frac{1}{\ln x}\Big|_3^b = \frac{1}{\ln 3} - \frac{1}{\ln b}$ (use the substitution $w = \ln x$.) This has limit $(1/\ln 3)$ as $b \to \infty$.

7.8, #27. $\displaystyle\int \frac{dz}{z^2 - z} = \ln|z-1| - \ln|z| + C = \ln(|z-1|/|z|) + C$ using partial fractions or formula 26 on page 367 with $x = z$, $a = 1$, $b = 0$. $f(z) = 1/(z^2 - z)$ has graph



and is undefined at $z = 0$ and $z = 1$. So $\int_a^b f(z)\,dz$ is improper if 0 or 1 is in $[a, b]$. $\ln|z-1| - \ln|z| + C$ has no limit as $z \to 0$ or as $z \to 1$ from either the right or the left. Thus $\int_a^x f(z)\,dz$ diverges if 0 or 1 is in $[a, x]$. In particular, $\int_1^3 f(z)\,dz$ diverges. If 0 and 1 are not in $[a, x]$, then $\int_a^x f(z)\,dz = \ln|x-1| - \ln|x| - \ln|a-1| + \ln|a| = \ln(|a||x-1|/|a-1||x|)$. Lastly,

$$\int_3^\infty \frac{dz}{z^2 - z}\,dz = \lim_{b\to\infty} \ln|1 - (1/b)| - \ln(2/3) = \ln(3/2).$$

7.8, #30. People get sick fastest when the rate $r = 1000te^{-0.5t}$ is maximal. Since $r' = 1000e^{-0.5t}(1 - .5t)$ is zero when $t = 2$, this happens 2 days after the start of the epidemic. The total number of people getting sick is 4000. Indeed,

$$\begin{aligned}\int_0^\infty 1000te^{-0.5t}\,dt &= \lim_{b\to\infty}\int_0^b 1000te^{-0.5t}\,dt\\ &= \lim_{b\to\infty} 1000(-2t-4)e^{-0.5t}\Big|_0^b\\ &= \lim_{b\to\infty} 1000((-2b-4)e^{-.5b}+4)\\ &= 4000\end{aligned}$$

7.9, #3–4. The integrals converge. Indeed,

$$\begin{aligned}\int_1^\infty \frac{dx}{x^3+1} &< \int_1^\infty \frac{dx}{x^3}\\ \int_1^\infty \frac{d\theta}{\sqrt{\theta^3+1}} &< \int_1^\infty \frac{d\theta}{\theta^{3/2}}\end{aligned}$$

and the integrals on the right converge since the integrands are of the form $1/x^p$ with $p > 1$.

7.9, #5. The integral diverges since

$$\int_1^b \frac{dx}{1+x} = \ln|1+x|\Big|_1^b = \ln|1+b| - \ln 2$$

diverges when $b \to \infty$.

7.9, #15. $\theta^3 + \theta$ behaves like θ as $\theta \to 0$ since $\theta^3 \leq \theta$ for $0 \leq \theta \leq 1$. So we would guess the integral converges. Indeed, $\theta^3 + \theta > \theta$, so

$$\int_0^1 \frac{d\theta}{\sqrt{\theta^3+\theta}} < \int_0^1 \frac{d\theta}{\sqrt{\theta}}.$$

The last integral converges since the integrand is of the form $1/\theta^p$ with $p < 1$.