7.6, #8. LEFT= 0.815997, RIGHT= 0.753497, and TRAP= 0.785724. The actual value of the integral is $\pi/4$. Here LEFT is an over estimate and RIGHT is an under estimate since the integrand $1/(1+x^2)$ is decreasing on $[0,1]$. Thus $0.753497 < \pi/4 < 0.815997$ and $3.013988 < \pi < 3.263988$.

7.6, #10. The function g is decreasing and concave-up on the interval $[0,1]$ of integration. So RIGHT<TRAP<LEFT since TRAP is the average of LEFT and RIGHT. We also have MID<TRAP since g is concave-up, and RIGHT<MID since g is decreasing. So LEFT= 0.664, TRAP= 0.633, MID= 0.632, and RIGHT= 0.601. The true value lies between TRAP and MID, that is, $0.633 > \int_0^1 g(t)\,dt > 0.632$

7.6, #21. It may help to think about this problem first in the case where 100000 is replaced by 4.
$\sum_{k=1}^{100000}(1/k)$ is a left sum for $\int_1^{100001}(1/x)\,dx$ with $N = 100000$. $\sum_{k=2}^{100000}(1/k)$ is a right sum for $\int_1^{100000}(1/x)\,dx$ with $N = 999999$. The integrand is decreasing, so

$$\sum_{k=1}^{100000}(1/k) \geq \int_1^{100001}\frac{dx}{x} = \ln 100001 = 11.51293546$$
$$\sum_{k=2}^{100000}(1/k) \leq \int_1^{100000}\frac{dx}{x} = \ln 100000 = 11.51292546$$

Thus $11.512 \leq \sum_{k=1}^{100000}(1/k) \leq 1 + 11.513$ (the term 1 comes from $1/k$ for $k = 1$). The nearest integer is 12.

7.7, #2. Simpson's rule with $n = 10$ gives the approximation 0.6931473747 for $\int_1^2(1/x)\,dx$. On the other hand, the integral is $\ln 2 = 0.6931471806$. The approximation is larger than the integral with the error being 0.0000001941.

7.7, #7. The approximations are 1.089429413 for $n = 10$, 1.089429413 for $n = 50$, 1.089429413 for $n = 250$. The approximations are the same to 9 decimal places, so we take 1.089 as the value of $\int_0^1 \sqrt{1+x^4}\,dx$.

7.7, #8. The approximations are 29.09408256 for $n = 10$, 29.09346052 for $n = 50$, 29.09346052 for $n = 250$. The last two approximations are the same to 8 decimal places, so we take 29.093 as the value of $\int_0^{10}\ln(z^2+1)\,dz$.