7.5, #46. The integrand is an improper rational function. Long division gives

$$\frac{x^2}{x^2+6x+13} = 1 - \frac{6x+13}{x^2+6x+13}.$$

Completing the square gives $x^2+6x+13 = (x+3)^2+2^2$, so set $w = x+3$ and $dw = dx$. Then

$$\frac{6x+13}{x^2+6x+13} = \frac{6w-5}{w^2+2^2},$$

so formula 25 applies with $a = 2$, $b = 6$, $c = -5$, giving

$$\int \frac{x^2}{x^2+6x+13}\,dx = x - 3\ln\left|(x+3)^2+2^2\right| + \frac{5}{2}\arctan\frac{x+3}{2} + C.$$

7.5, #50. The integrand is an improper rational function. Long division gives

$$\frac{t^2+1}{t^2-1} = 1 + \frac{2}{t^2-1}.$$

Since $t^2-1 = (t-1)(t+1)$, formula 26 with $a = 1$, $b = -1$ gives

$$\int \frac{t^2+1}{t^2-1}\,dt = t + \ln|t-1| - \ln|t+1| + C.$$

7.5, #51. Formula 24 with $a = 2$ gives

$$\int_0^2 \frac{dx}{4+x^2} = \frac{1}{2}\arctan\frac{x}{2}\Big|_0^2 = \frac{1}{2}(\arctan 1 - \arctan 0) = \frac{\pi}{8} = 0.3927.$$

The left and right Riemann sums with $n = 100$ are approximately 0.3939 and 0.3915. Since the integrand is decreasing on the interval $[0,2]$, its value is between these two estimates.

7.5, #53. Since $x^2+2x+5=(x+1)^2+2^2$, formula 24 with $a=2$ gives

$$\int_0^1 \frac{dx}{x^2+2x+5} = \frac{1}{2}\arctan\frac{x+1}{2}\bigg|_0^1 = \frac{1}{2}(\arctan 1 - \arctan\frac{1}{2}) = 0.1609.$$

The left and right Riemann sums with $n=100$ are approximately 0.1613 and 0.1605. Since the integrand is decreasing on the interval $[0,1]$, its value is between these two estimates.

7.5, #64. (a) The average voltage over a second is

$$\frac{1}{1-0}\int_0^1 V_0\cos(120\pi t)\,dt = \frac{V_0}{120\pi}\sin(120\pi t)\bigg|_0^1 = 0.$$

(b) The average of $V^2$ over a second is

$$\begin{aligned}\frac{1}{1-0}\int_0^1 V_0^2\cos^2(120\pi t)\,dt &= \frac{V_0^2}{120\pi}\int_0^{120\pi}\cos^2 u\,du \\ &= \frac{V_0^2}{240\pi}(\cos u\sin u+u)\bigg|_0^{120\pi} = \frac{1}{2}V_0^2\end{aligned}$$

using the substitution $u=120\pi t$ and $du=120\pi\,dt$. Thus $\overline{V}=V_0/\sqrt{2}$.

(c) If $\overline{V}=110$, then $V_0=110\sqrt{2}=155.56$ volts.

7.6, #1.

| | $n=1$ | $n=2$ | $n=4$ |
|---|---|---|---|
| LEFT | 40.0000 | 40.7846 | 41.7116 |
| RIGHT | 51.2250 | 46.3971 | 44.5179 |
| TRAP | 45.6125 | 43.5909 | 43.1147 |
| MID | 41.5692 | 42.6386 | 42.8795 |

7.6, #3.

| $n$ | 10 | 100 | 1000 |
|---|---|---|---|
| LEFT | 5.4711 | 5.8116 | 5.8464 |
| RIGHT | 6.2443 | 5.8890 | 5.8541 |
| TRAP | 5.8577 | 5.8503 | 5.8502 |
| MID | 5.8465 | 5.8502 | 5.8502 |

$e^x\sqrt{x}$ is increasing and concave-up on $[1,2]$ since

$$(e^x\sqrt{x})' = e^x\left(\sqrt{x}+\frac{1}{2\sqrt{x}}\right), \quad (e^x\sqrt{x})'' = e^x\left(\sqrt{x}+\frac{1}{\sqrt{x}}-\frac{1}{4\sqrt{x^3}}\right)$$

are positive on $[1,2]$. ($\sqrt{x}\geq 1$ and $1/(4\sqrt{x^3})\leq 1/4$ for $1\leq x\leq 2$.) So LEFT and MID underestimate the integral, RIGHT and TRAP overestimate the integral.

7.6, #4.

| $n$ | 10 | 100 | 1000 |
|---|---|---|---|
| LEFT | 3.0132 | 2.9948 | 2.9930 |
| RIGHT | 2.9711 | 2.9906 | 2.9925 |
| TRAP | 2.9922 | 2.9927 | 2.9927 |
| MID | 2.9930 | 2.9927 | 2.9927 |

$\sqrt{3+\cos\theta}$ is decreasing and concave-down on $[0,\pi/2]$ since

$$(\sqrt{3+\cos\theta})' = \frac{-\sin\theta}{2\sqrt{3+\cos\theta}},$$

$$(\sqrt{3+\cos\theta})'' = \frac{2(3+\cos\theta)^{1/2}(-\cos\theta)+\sin\theta(3+\cos\theta)^{-1/2}(-\sin\theta)}{4(3+\cos\theta)}$$

are negative on $[0,\pi/2]$. (The second derivative is messy, but its sign is easy to determine. The numerator is always negative since square roots are non-negative; the denominator is positive for $0\leq\theta\leq\pi/2$.) So LEFT and MID overestimate the integral, RIGHT and TRAP underestimate the integral.