

7.5, #17. Formula 15 with $x = y$, $p(x) = y^2$, $a = 2$ gives

$$\int y^2 \sin 2y \, dy = -\frac{1}{2}y^2 \cos 2y + \frac{1}{4}2y \sin 2y + \frac{1}{8}2 \cos 2y + C.$$

7.5, #18. Formula 11 with $x = y$, $a = 2$, $b = 7$ gives

$$\int \cos 2y \cos 7y \, dy = \frac{1}{45}(7 \cos 2y \sin 7y - 2 \sin 2y \cos 7y) + C.$$

7.5, #24. Substitute $x = 3y$, $dx = 3dy$ to get $\int \cos^4 3y \, dy = (1/3) \int \cos^4 x \, dx$. Then use formula 18 with $n = 4$ to get

$$\int \cos^4 x \, dx = \frac{1}{4} \cos^3 x \sin x + \frac{3}{4} \int \cos^2 x \, dx$$

Use formula 18 again with $n = 2$ to get

$$\int \cos^2 x \, dx = \frac{1}{2} \cos x \sin x + \int dx = \frac{1}{2} \cos x \sin x + \frac{1}{2}x + C.$$

Putting these pieces together gives

$$\int \cos^4 3y \, dy = \frac{1}{12} \cos^3 3y \sin 3y + \frac{1}{8} \cos 3y \sin 3y + \frac{3}{8}y + C.$$

7.5, #28. Make the Substitution $w = 2x$, $dw = 2dx$ and then use formula 24 to get

$$\int \frac{dx}{1+4x^2} = \frac{1}{2} \int \frac{dw}{1+w^2} = \frac{1}{2} \arctan w + C = \frac{1}{2} \arctan 2x + C.$$

Comment: A very common mistake — using formula 24 without the first making the substitution $w = 2x$. This gives incorrect answer $\arctan 2x + C$.

7.5, #37. Formula 26 with $x = z$, $a = 3$, and $b = 0$ gives

$$\int \frac{dz}{z(z-3)} = \frac{1}{3}(\ln |z-3| - \ln |z|) + C.$$

7.5, #38. Formula 26 with $x = y$, $a = 2$, and $b = -2$ gives

$$\int \frac{dy}{4-y^2} \, dy = - \int \frac{dy}{(y-2)(y+2)} = -\frac{1}{4}(\ln |y-2| - \ln |y+2|) + C.$$

7.5, #44. Since $y^2 + 4y + 5 = (y+2)^2 + 1^2$, we substitute $w = y+2$ and $dw = dy$ to get

$$\int \frac{dy}{y^2 + 4y + 5} = \int \frac{dw}{w^2 + 1^2} = \arctan w + C = \arctan(y+2) + C.$$

7.5, #45. The integrand is an improper rational function, so we use long division to write

$$\frac{x^3 + 3}{x^2 - 3x + 2} = x + 3 + \frac{7x - 3}{x^2 - 3x + 2}.$$

Now $x^2 - 3x + 2 = (x-1)(x-2)$, so formula 27 with $a = 1$, $b = 2$, $c = 7$, and $d = -3$ applies, giving

$$\int \frac{x^3 + 3}{x^2 - 3x + 2} \, dx = \frac{1}{2}x^2 + 3x - 4 \ln |x-1| + 11 \ln |x-2| + C.$$