7.3, #17. Set $w = x^2 + 4$, $dw = 2x\,dx$. Then $w = 20$ if $x = 4$, $w = 5$ if $x = 1$, and

$$\int_4^1 x\sqrt{x^2+4}\,dx = \frac{1}{2}\int_{20}^5 w^{1/2}\,dw = \frac{1}{3}w^{3/2}\Big|_{20}^5 = \frac{1}{3}(5^{3/2} - 20^{3/2}) = -26.087.$$

7.3, #20. Set $w = x^2 + 4x + 5$, $dw = (2x+4)\,dx$. Then $w = 1$ if $x = -2$, $w = 5$ if $x = 0$, and

$$\int_{-2}^0 \frac{2x+4}{x^2+4x+5}\,dx = \int_1^5 \frac{dw}{w} = \ln 5 - \ln 1 = \ln 5 = 1.609.$$

7.3, #27. $x^2 + 4x + 5 = (x^2 + 4x + 4) + 1 = (x+2)^2 + 1$. Set $x + 2 = \tan\theta$, $dx = \sec^2\theta\,d\theta$. Then

$$\int \frac{dx}{x^2+4x+5} = \int \frac{dx}{(x+2)^2+1} = \int \frac{\sec^2\theta}{\tan^2\theta+1}\,d\theta$$
$$= \int d\theta = \theta + C = \arctan(x+2) + C$$

using the trigonometric identity $\tan^2\theta + 1 = \sec^2\theta$.

7.3, #29. The area is $\int_0^2 xe^{x^2}\,dx$. We evaluate this by substitition. Let $u = x^2$. Then $du = 2x\,dx$. Thus the area is

$$\int_0^2 xe^{x^2}\,dx = \frac{1}{2}\int_0^2 2xe^{x^2}\,dx = \frac{1}{2}\int_{0^2}^{2^2} e^u\,du = \frac{1}{2}(e^4 - e^0) = \frac{1}{2}(e^4 - 1) \simeq 26.7991.$$

7.4, #3. Set $te^{5t} = uv'$, where $u = t$, $v' = e^{5t}$. Then $u' = 1$, $v = (1/5)e^{5t}$, and

$$\int te^{5t}\,dt = \frac{1}{5}te^{5t} - \frac{1}{5}\int e^{5t}\,dt = \frac{1}{5}te^{5t} - \frac{1}{25}e^{5t} + C.$$

7.4, #4. Set $t^2e^{5t} = uv'$, where $u = t^2$, $v' = e^{5t}$. Then $u' = 2t$, $v = (1/5)e^{5t}$, and

$$\int t^2e^{5t}\,dt = \frac{1}{5}t^2e^{5t} - \frac{2}{5}\int te^{5t}\,dt = \frac{1}{5}t^2e^{5t} - \frac{2}{5}\left[\frac{1}{5}te^{5t} - \frac{1}{25}e^{5t}\right] + C$$
$$= \frac{1}{5}t^2e^{5t} - \frac{2}{25}te^{5t} + \frac{2}{125}e^{5t} + C$$

7.4, #6. Set $y\ln y = uv'$, where $u = \ln y$, $v' = y$. Then $u' = 1/y$, $v = y^2/2$, and

$$\int y\ln y\,dy = \frac{1}{2}y^2\ln y - \frac{1}{2}\int \frac{y^2}{y}\,dy = \frac{1}{2}y^2\ln y - \frac{1}{4}y^2 + C.$$

7.4, #7. Set $x^3\ln x = uv'$, where $u = \ln x$, $v' = x^3$. Then $u' = 1/x$, $v = x^4/4$, and

$$\int x^3\ln x\,dx = \frac{x^4\ln x}{4} - \frac{1}{4}\int \frac{x^4}{x}\,dx = \frac{x^4\ln x}{4} - \frac{x^4}{16} + C.$$