7.2, #12. Set $w = -x^2$, $dw = -2x\,dx$, then

$$\int xe^{-x^2}\,dx = -\frac{1}{2}\int e^w\,dw = -\frac{1}{2}e^w + C = -\frac{1}{2}e^{-x^2} + C.$$

7.2, #16. Set $w = y + 5$, $dw = dy$, then

$$\int \frac{1}{y+5}\,dx = \int \frac{1}{w}\,dw = \ln w = \ln(y+5).$$

7.2, #20. Set $w = 4 - x$, $dw = -dx$, then

$$\int \frac{dx}{\sqrt{4-x}} = -\int w^{-1/2}\,dw = -2w^{\frac{1}{2}} + C = -2\sqrt{4-x} + C.$$

7.2, #24. Set $w = \cos 3t$, $dw = -3\sin 3t\,dt$, then

$$\int \sqrt{\cos 3t}\sin 3t\,dt = -\frac{1}{3}\int w^{1/2}\,dw = -\frac{2}{9}w^{3/2} + C = -\frac{2}{9}(\cos^{3/2} 3t) + C.$$

7.2, #28. Set $w = \sqrt{x}$, $dw = 1/(2\sqrt{x})\,dx$, then

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = 2\int \cos w\,dw = 2\sin w + C = 2\sin\sqrt{x} + C.$$

7.3, #1(a). Set $w = 1 + x^2$, $dw = 2x\,dx$. Then $w = 1$ if $x = 0$, $w = 2$ if $x = 1$, and

$$\int_0^1 \frac{x}{1+x^2}\,dx = \frac{1}{2}\int_1^2 \frac{dw}{w} = \frac{1}{2}\ln|w|\Big|_1^2 = \frac{1}{2}\ln 2 = 0.347$$

#1(b). Set $w = \cos x$, $dw = -\sin x\,dx$. Then $w = 1$ if $x = 0$, $w = 1/\sqrt{2}$ if $x = \pi/4$, and

$$\int_0^{\pi/4} \frac{\sin x}{\cos x}\,dx = -\int_1^{1/\sqrt{2}} \frac{dw}{w} = -\ln|w|\Big|_1^{1/\sqrt{2}} = -\ln(1/\sqrt{2}) = 0.347.$$

The answers are the same since $-\ln(1/\sqrt{2}) = \ln\sqrt{2} = (1/2)\ln 2$.

7.3, #2. Set $w = \pi x$, $dw = \pi dx$. Then $w = 0$ if $x = 0$, $w = \pi/2$ if $x = 1/2$, and

$$\int_0^{1/2} \cos\pi x\,dx = \frac{1}{\pi}\int_0^{\pi/2} \cos w\,dw = \frac{1}{\pi}\sin w\Big|_0^{\pi/2} = \frac{1}{\pi} = 0.318.$$

7.3, #3. Set $w = x^{1/3}$, $dw = (1/3)x^{-2/3}\,dx$. Then $w = 1$ if $x = 1$, $w = 2$ if $x = 8$, and

$$\int_1^8 \frac{e^{\sqrt[3]{x}}}{\sqrt[3]{x^2}}\,dx = 3\int_1^2 e^w\,dw = 3e^w\Big|_1^2 = 3(e^2 - e) = 14.012.$$