6.4, #22. Noting that the derivative of $\sin(t^2)$ is $2t\cos(t^2)$ we have that $\sin(t^2)$ is an antiderivative of $2t\cos(t^2)$. Thus $\displaystyle\int 2t\cos(t^2)\,dx = \sin(t^2) + C$.

6.4, #23. See the previous problem. From it we see that the derivative of $\frac{1}{2}\sin(t^2)$ is $t\cos(t^2)$. Thus $\frac{1}{2}\sin(t^2)$ is an antiderivative of $t\cos(t^2)$. Observe that $\displaystyle\int t\cos(t^2)\,dx = \frac{1}{2}\sin(t^2) + C$.

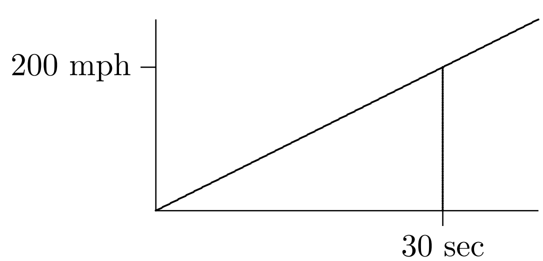
6.4, #30. An antiderivative of $F'(x) = e^x$ has the form $F(x) = e^x + C$. The condition $F(0) = 2$ translates to $1 + C = 2$. Thus $C = 1$ and $F(x) = e^x + 1$.

6.4, #31. An antiderivative of x^2 has the form $F(x) = \frac{1}{3}x^3 + C$. The condition $F(0) = 2$ translates to $0 + C = 2$. Thus $C = 2$ and $F(x) = \frac{1}{3}x^3 + 2$.

6.4, #32. An antiderivative of $F'(x) = \cos x$ has the form $F(x) = \sin x + C$. The condition $F(0) = 2$ translates to $\sin 0 + C = 2$. Thus $C = 2$ and $F(x) = \sin x + 2$.

6.4, #50. Acceleration is constant, say a miles/hr^2. The plane has initial velocity $v_0 = 0$ mph and initial position $s_0 = 0$. So the velocity and position functions of the plane on the runway are $v(t) = at$ and $s(t) = (1/2)at^2$. The plane reaches velocity 200 mi/hr in 30 seconds, that is, $200 = v(30/3600) = a \times (30/3600)$. Thus $a = 24000$ mi/hr^2. Since $s(30/3600) = (1/2) \times 24000 \times (30/3600)^2 = 5/6$ miles, the length of the runway is at least 5/6 miles.

A shorter proof: The problem is to compute the distance the plane has traveled when its velocity is 200 mi/hr. The graph of velocity as a function of time is



The desired distance is the area of the triangle formed by the graph on the interval $[0, 30]$. So the distance is $(1/2)(200)(30)(1/3600) = 0.833$ miles.

6.4, #54. The height of the rock t seconds after being thrown is $s(t) = v_0 t - (1/2)gt^2$ feet, where $s(t)$ is measured from the initial position of the rock, v_0 is the initial velocity of the rock, and $g = 32$ ft/sec^2. The rock reaches its highest height h when $s'(t) = 0$, that is, when $t = v_0/g$. Thus $h = s(v_0/g) = v_0(v_0/g) - (1/2)g(v_0/g)^2$ and $v_0^2 = 2hg$. In part (a) $h = 100$, so $v_0 = \sqrt{200 \times 32} = 80$ ft/sec. In part (b) $g = 5$ and $v_0 = 80$, so $h = (80)^2/(2 \times 5) = 640$ feet.

6.Rev, #36. $v = -32t + v_0 = -32t + 160$. We plot v (vertical axis) versus t (horizontal axis). The graph of v is a straight line with slope -32 and v-intercept 160.

a) The maximum height occurs when $v = 0$. Thus $32t = 160$ which means that $t = 5$.

b) The object hits the ground when the area above the t-axis is equal to the area below the t-axis. This happens when $t = 10$.

c) The maximum height is the area bounded by the graph of $v = -32t + 160$, the t-axis and the vertical lines $t = 0$ and $t = 5$. This region is a triangle with base of length 5 and height 160. Thus the maximum height, or area, is $\frac{160 \times 5}{2} = 80 \times 5 = 400$ feet per second.

d) By antidifferentiation we compute the greatest height by

$$\int_0^5 v\,dt = \int_0^5 (-32t + 160)\,dt = (-32(\frac{t^2}{2}) + 160t)|_0^5$$

$$= -32(\frac{5^2}{2}) + 160(5) - 0 = 16(5)(-5 + 10) = 16(5)(5) = 400$$

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6.Rev, #42. The height H increases until it reaches the top of the inverted "vee". Say this happens for time $0 \le t \le T$. Then H is constant for time $T \le t \le 2T$ as water fills the corresponding space on the right. H increases again for $t \ge 2T$, but at a slower rate than before since the space being filled corresponds to the entire width of the container. Note too that the container widens from bottom to top, so the increasing portions of the graph are concave-down.