6.1, #1. The left and right sums with four subdivisions for $\int_1^2 x^2\,dx$ are

$$LS = (1.00)^2(.25) + (1.25)^2(.25) + (1.50)^2(.25) + (1.75)^2(.25) = 1.96875$$
$$RS = (1.25)^2(.25) + (1.50)^2(.25) + (1.75)^2(.25) + (2.00)^2(.25) = 2.71875$$

Since $x^2$ is increasing on $[1,2]$, it follows that $LS \le \int_1^2 x^2\,dx \le RS$. Moreover, $RS - LS = (2^2 - 1^2)(.25) = 0.75$. In particular, the average $(1/2)(LS + RS) = 2.34375$ is within $(1/2)(0.75) = 0.375$ of the actual value.

6.1, #2. Since $f(x)$ is positive on $[1,6]$, the integral $\int_1^6 f(x)\,dx$ is the area bounded between the graph of $f$, the $x$-axis, and the lines $x = 1$, $x = 6$. So the area is 8.5. The average value of $f$ on $[1,6]$ is

$$\frac{1}{6-1}\int_1^6 f(x)\,dx = \frac{8.5}{5} = 1.7$$

6.1, #3. The average value of $f(x) = 4x + 7$ on $[1,3]$ is

$$\frac{1}{3-1}\int_1^3 (4x+7)\,dx = \frac{1}{2}(2x^2+7x)\Big|_1^3 = 15.$$

6.1, #5.

$$\int_0^2 (3x^2+1)\,dx = (x^3+x)\Big|_0^2 = 10.$$

6.1 #6. Since $-1 \le \sin x \le 1$ and $e^{-x} > 0$ for all $x$, the graph of $y = e^{-x}\sin x$, $x \ge 0$, lies between the graphs of $y = e^{-x}$ and $y = -e^{-x}$. The effect of multiplying $\sin x$ by $e^{-x}$ is to "dampen" the graph of $y = \sin x$. Now the graph of $y = \sin x$ on $[0,\pi]$ is above the $x$-axis. The portion of the graph on $[\pi, 2\pi]$ is the mirror image of that just described and is below the $x$-axis. Since $y = e^{-x}$ is a decreasing function we now conclude that $\int_0^{2\pi} e^{-x}\sin x dx > 0$.